Advanced Algebra
Unit 8 Review

Name: key
Period: ____

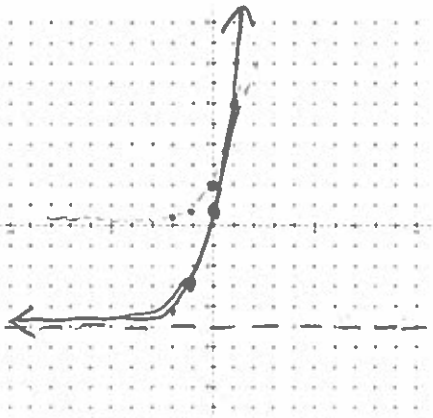
Part 1: No Calculator

Graph the function.

1. $y = 2(3)^{x+1} - 5$ Left 1 Down 5

$y = 2 \cdot (3)^x$

X	Y
-2	2/9
-1	2/3
0	2
1	6
2	18



Domain: $\mathbb{R}$ Reals

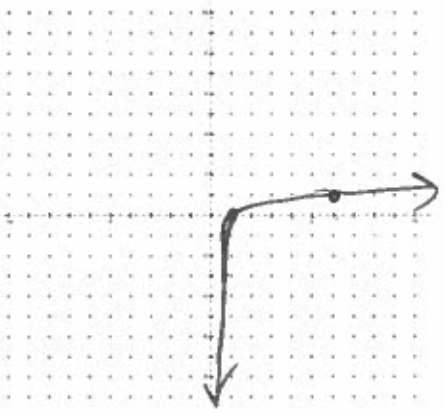
Range: $y > -5$

Asymptote: $y = -5$

2. $y = \log_6 x$

$6^y = x$

X	Y
1/36	-2
1/6	-1
1	0
6	1
36	2



Domain: $x > 0$

Range: $\mathbb{R}$

Asymptote: $x = 0$

3. Use $y = (.92)^x$ to answer the following:

a. What is the starting amount? 1

b. Is this a Exp Growth or Exp Decay? Why? Exp Decay $0 < b < 1$

c. What is the rate/percent? 8%

d. Rewrite as a log function: $\log_{.92} y = x$

4. Rewrite the expression in exponential form.

a. $\log_{16} 4 = \frac{1}{2}$ $16^{\frac{1}{2}} = 4$

b. $\ln x = 3$ $e^3 = x$

5. Expand the expressions

a. $\log\frac{2x^3}{5}$

$\log 2 + 3\log x - \log 5$

b. $\ln\frac{3\sqrt{x}}{y^5 z}$

$\ln 3 + \frac{1}{2}\ln x - 5\ln y - \ln z$

6. Condense the expressions

a. $\log_3 4 + \log_3 2 + \log_3 2$

$\log_3 4 \cdot 2 \cdot 2 = \log_3 16$

b. $\log 3 + \frac{1}{2}\log x - \log 5$

$\log \frac{3\sqrt{x}}{5}$

7. Evaluate or solve for x.

a. $\log_5 125$

$5^x = 125$

$\boxed{3}$

b. $\log_{36} 6$

$36^x = 6$

$\boxed{\frac{1}{2}}$

c. $\log_4 1$

$4^x = 1$

$\boxed{0}$

d. $\log_3 \frac{1}{27}$

$3^x = \frac{1}{27}$

$\boxed{-3}$

e. $\ln e^{12}$

$e^x = e^{12}$

$\boxed{12}$

f. $\log_5 x = -2$

$3^{-2} = x$

$\boxed{\frac{1}{9}}$

g. $\log_{\frac{1}{3}} x = 3$

$\left(\frac{1}{3}\right)^3 = x$

$\boxed{\frac{1}{27}}$

8. Explain the difference between a common logarithm and a natural logarithm.

Common log has a base of 10 ($\log = \log_{10}$)

Natural log has a base of e ($\ln = \log_e$)

Part 2: Calculator

9. From 1990 to 2000, the population of California can be modeled by $P = 27{,}216{,}000(1.0228)^t$ where t is the number of years since 1990.

a. Estimate the population in 2004. $\approx 37{,}315{,}938$ people

$t = 14$

b. When will California reach 40,000,000? 17.08 years = 2007

$40{,}000{,}000 = 27{,}216{,}000$

c. Will this trend continue forever? Explain. No. At some point the growth factor will change.

10. You buy a new car for $22,500. The value of the car decreases by 25% each year.

a. Write an exponential model giving the car's value V (in dollars) after t years. $V = 22500(.75)^t$

b. What is the value of the car after 3 years? $9492.19

c. In how many years is the car worth $5300 (must show algebraic work and be accurate to 2 decimal places)?

$5300 = 22500(.75)^t$

$\frac{5300}{22500} = .75^t$

$\log_{.75}\left(\frac{5300}{22500}\right) = t$

$t = \frac{\log\left(\frac{5300}{22500}\right)}{\log .75}$

$t \approx 5.03$ years

11. You deposit $2200 in an account that pays 3% annual interest. Find the balance after 15 years if the interest is compounded with the given frequency. Show your set-up!!!!!!!!!!

a. quarterly

$y = 2200\left(1 + \frac{.03}{4}\right)^{4 \cdot 15}$

$3444.50

b. continuously

$y = 2200 \cdot e^{.03 \cdot 15}$

$3450.29

12. Your goal is to have $11,000 to buy a used car in 2 years. How much would you need to deposit today if your account pays 4.5% annual interest, compounded monthly?

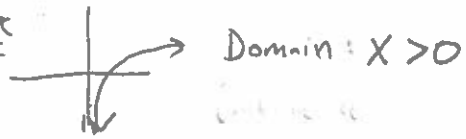
$11000 = P\left(1 + \frac{.045}{12}\right)^{2 \cdot 12}$ $\quad P = \frac{11000}{\left(1 + \frac{.045}{12}\right)^{24}} = \$10{,}054.94$

13. What is *e* approximately equal to (accurate to three decimal places)? Is *e* rational or irrational? Explain.

$e \approx 2.718$ Irrational, "e" goes on forever and can't be written as a fraction.

14. Why is $\log_2(-6)$ not possible? Why can't you take a logarithm of a negative number? Use complete sentences.

A positive base to any power is always a positive answer. $2^x \neq -6$ OR



15. Evaluate the logarithm. Round answer to 3 decimal places

a. $\log_5 1.25$ $\quad \frac{\log 1.25}{\log 5} = .139$

b. $\log_{\frac{1}{3}} 0.0005$ $\quad \frac{\log .0005}{\log \frac{1}{3}} = 6.919$

c. $\ln 24$ $\quad 3.178$

16. Solve the exponential equation. Check for extraneous solutions. Round the result to 3 decimal places if necessary.

a. $3e^{3x} = 12$

$e^{3x} = 4$

$\log_e 4 = 3x$ $\quad \ln 4 = 3x$

$x \approx .462$

b. $10^{-x+4} + 7 = 5$

$10^{-x+4} = -2$ $\quad$ No Solution

$\log_{10} -2 = -x + 4$ $\quad$ Can't take the log of a negative #

c. $9^{2x} = 3^{2x+4}$

$3^{2 \cdot 2x} = 3^{2x+4}$

$4x = 2x + 4$

$2x = 4$ $\quad x = 2$

d. $5^{0.5x} + 12 = 21$

$5^{.5x} = 9$

$\log_5 9 = .5x$ $\quad x \approx 2.730$

17. Solve the logarithmic equation. Check for extraneous solutions. Round the result to 3 decimal places if necessary.

a. $\log_2(3x-1)=8$

$2^8 = 3x-1$

$256 = 3x-1$

$3x = 257$

$x = \frac{257}{3} \approx 85.667$

b. $\ln(3x-3)=\ln(x+15)$

$3x-3 = x+15$

$2x = 18$

$x = 9$

c. $4+\log_9(3x-7)=6$

$\log_9(3x-7)=2$

$9^2 = 3x-7$

$81 = 3x-7$

$x = \frac{88}{3} \approx 29.333$

d. $\log_4 x+\log_4(5x-2)=2$

$5x$	8
x	-2

$\log_4(5x^2-2x)=2$

$4^2 = 5x^2-2x$

$5x^2-2x-16=0$

$(5x+8)(x-2)=0$

$x = -\frac{8}{5}$

$x = 2$

18. The pH of a patient's blood can be calculated using the Henderson-Hasselbach Formula, $pH = 6.1+\log\frac{B}{C}$, where B is the concentration of bicarbonate and C is the concentration of carbonic acid. The normal pH of blood is approximately 7.4.

a. Expand the right side of the formula.

$pH = 6.1 + \log B - \log C$

b. Find the pH of blood that has bicarbonate concentration of 38 and carbonic acid concentration of 2

$pH = 6.1 + \log 38 - \log 2$ OR $6.1 + \log\left(\frac{38}{2}\right)$

$pH \approx 7.379$