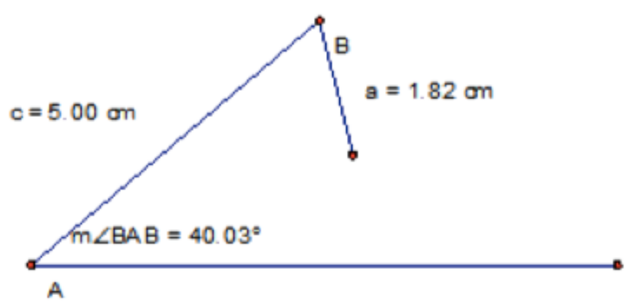
Ambiguous SSA Triangles Enrichment

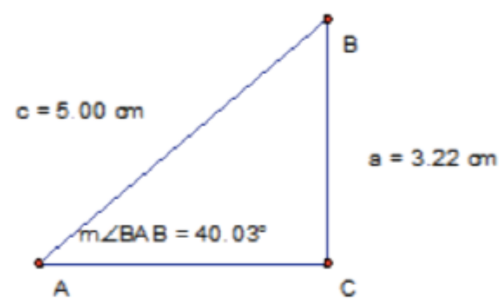
In Geometry you were never allowed to use SSA to prove congruent triangles. The triangles may have been congruent but there are some cases where there could be 2 triangles so the information was inconclusive to whether they were congruent. In this lesson you will determine how many triangles can result from the given SSA dimensions and you will find the dimensions to both triangles when there are 2 that result.

For consistency we will always be given angle A, side a, and side c. We could use the pattern of this relationship to work with other SSA problems.

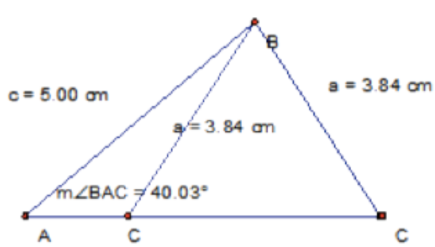
Below are 4 cases where A=40^0, c=5cm and "a" varies to demonstrate the 4 situations that we will examine. The length of the side opposite of the given angle determines how many triangles will result.



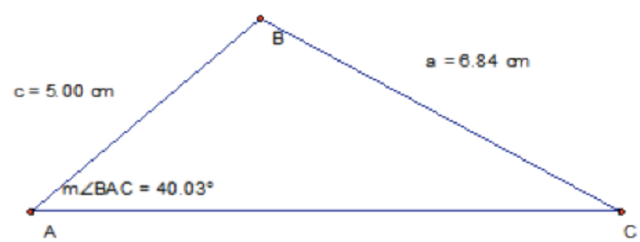
"a" is too short to form a triangle



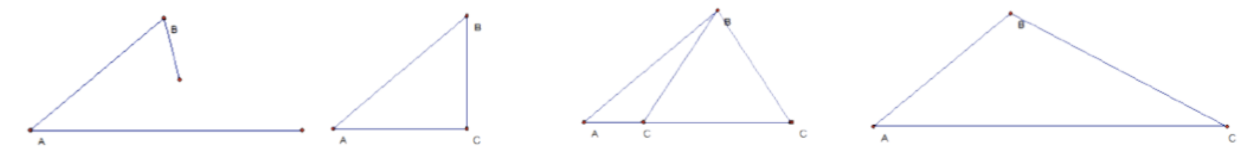
"a" is the height



"a" is longer and can go either direction to make 2 triangles



"a" is longer than "c"



0 triangles	1 right triangle	2 triangles	1 oblique triangle
$a < c \sin A$	$a = c \sin A$	$c \sin A < a < c$	$a \geq c$

height

Example:

How many triangles exist with the dimensions: $A = 50^0$, $a = 57$, $c = 68$

First, it is not an oblique triangle because $57 \not> 68$.

Find height which is "c sinA".

$h = 68 \sin 50^0 = 52.09$

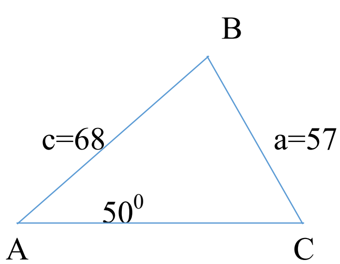
57>52.09 so there are 2 triangles. If "a" was smaller than 52.09, there would be no triangles and if it was the same as 52.09, then there would be one triangle.

1. How many triangles exist? Show why.

a.)	b.)	c.)	d.)
$A = 40^0$	$A = 25^0$	$A = 70^0$	$A = 30^0$
$a = 8$	$a = 2$	$a = 24$	$a = 3$
$c = 6$	$c = 7$	$c = 25$	$c = 6$

Example: Solving 2 triangles

$A = 50^0$, $a = 57$, $c = 68$



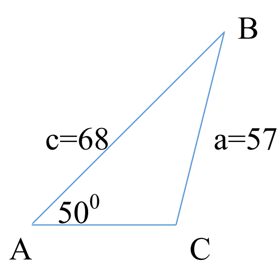
$$\frac{\sin 50^\circ}{57} = \frac{\sin C_1}{68}$$

$$C_1 = 66.0^\circ$$

$$B_1 = 180^\circ - 50^\circ - 66^\circ = 64^\circ$$

$$\frac{\sin 50^\circ}{57} = \frac{\sin 64}{b_1}$$

$$b_1 = 66.9$$



To find the other C:

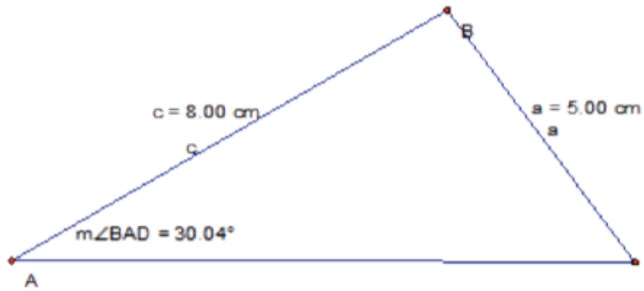
$C_2 = 180^0 - C_1 = 180^0 - 66.0^0 = 114^0$

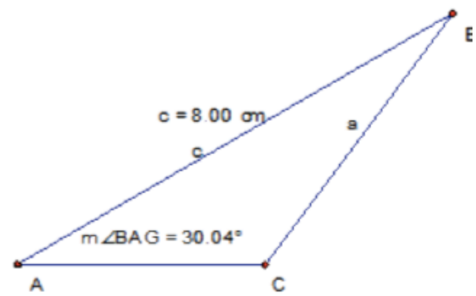
$B_2 = 180^0 - 50^0 - 114^0 = 16^0$

$$\frac{\sin 50^\circ}{57} = \frac{\sin 16^\circ}{b_2}$$

$$b_2 = 20.5$$

2. Solve both triangles
A= 30^0, a=5cm, c=8cm





3. Give an example that would result in the following: (make A<90^0 in each case) Hint: you may choose to keep the same angle A for all. Examples need to be different than what has been used in this lesson.

a.) 1 right triangle	b.) 1 oblique triangle	c.) 2 triangles	d.) no triangles
A=	A=	A=	A=
a=	a=	a=	a=
c=	c=	c=	c=