9.2 Add and Subtract Rational Expressions

Key

- I can find a common denominator.
- I can add and subtract rational expressions.

To add or subtract rational expressions you must have _Common · denominators_ !!

Add with like denominators

Ex 1: $\dfrac{9}{6x} + \dfrac{2}{6x} = \dfrac{11}{6x}$

Ex 2: $\dfrac{5}{2x+5} - \dfrac{x+4}{2x+5}$

$$\dfrac{5-(x+4)}{2x+5} = \dfrac{-x+1}{2x+5}$$

Finding a Least Common Denominator (LCD)

**Factor, include all factors but don't repeat any factors that are already common

Find the LCD for the following denominators.

Ex 3: $5x$, x^2+5x

$5 \cdot x \qquad x(x+5)$

$5x(x+5)$

Ex 4: x^2-4, $3x-6$, $3x^2$

$(x-2)(x+2) \qquad 3(x-2) \qquad 3x^2$

$3x^2(x-2)(x+2)$

Do & Discuss:

1. Are these denominators the same?

a. $\dfrac{5}{5+x} + \dfrac{4x}{x+5}$ yes

b. $\dfrac{3+x}{6-x} + \dfrac{7}{x-6}$ no

c. $\dfrac{5}{4x} + \dfrac{9x}{x+4}$ no

2. Find the LCD
x^2-5x-6, $2x+2$, $x^2-11x+30$

$(x-6)(x+1)$, $\quad 2(x+1)$, $\quad (x-5)(x-6)$

$2(x-6)(x+1)(x-5)$

Add and Subtract with unlike denominators

EX 3: $\dfrac{5}{4x^2} + \dfrac{x+1}{2x^2+4x}$

$\dfrac{(x+2)}{(x+2)}\dfrac{5}{4x^2} + \dfrac{x+1}{2x(x+2)}\cdot\dfrac{2x}{2x}$

$\dfrac{5x+10}{4x^2(x+2)} + \dfrac{2x^2+2x}{4x^2(x+2)} = \boxed{\dfrac{2x^2+7x+10}{4x^2(x+2)}}$

← doesn't factor
so won't simplify any more

Steps:
1. Factor denominator
2. Find the LCD (least common denominator)
3. Multiply numerator and denominator to make all denominators common.
4. Add/subtract numerator, use same denominator.
5. Simplify (factor numerator and cancel common factors)

Ex 4: $\dfrac{7}{3x-9} - \dfrac{x+4}{x^2-9}$

$\dfrac{(x+3)}{(x+3)}\dfrac{7}{3(x-3)} - \dfrac{x+4}{(x-3)(x+3)}\cdot\dfrac{3}{3}$

$\dfrac{7x+21}{3(x-3)(x+3)} + \dfrac{-3x-12}{3(x-3)(x+3)} = \boxed{\dfrac{4x+9}{3(x-3)(x+3)}}$

doesn't factor
so won't simplify any more

✱ when subtracting distribute negative to numerator!!

Perform the indicated operation and simplify

4. $\dfrac{x+1}{x^2+6x+9}\dfrac{(x-3)}{} + \dfrac{6}{x^2-9}\dfrac{(x+3)}{}$

$\dfrac{(x+3)(x+3)(x-3)}{}\quad \dfrac{(x+3)(x-3)}{}\ (x+3$

$\dfrac{x^2-2x-3 + 6x+18}{(x+3)^2(x-3)}$

$\boxed{\dfrac{x^2+4x+15}{(x+3)^2(x-3)}}$

5. $\dfrac{8}{4x^2}\dfrac{(2x-3)}{} - \dfrac{2+x}{8x^2-12x}\cdot\dfrac{x}{x}$

$\dfrac{2x-3}{}\qquad 4x(2x-3)\cdot x$

$\dfrac{16x-24 - 2x - x^2}{4x^2(2x-3)}$

$\boxed{\dfrac{-x^2+14x-24}{4x^2(2x-3)}}$

Additional resources:

- Textbook Section 8.5 (pg. 582)
- https://www.khanacademy.org/math/algebra2/rational-expressions-equations-and-functions/adding-and-subtracting-rational-expressions/v/adding-rational-expression-w-unlike-denominators