Key

8.6: Solving Exponential and Logarithmic Equations

- I can solve a given exponential or logarithmic equation
- I can write an equation to model an exponential situation and solve the equation

Properties of Equality

Exponential Equations: $b^x = b^y$ if and only if $x = y$. Ex: If $3^{2x} = 3^{10}$, then $2x = 10$, $x = 5$.

Logarithmic Equations: If $\log_b x = \log_b y$ if and only if $x = y$. If $\log_5 \frac{x}{2} = \log_5 7$, then $\frac{x}{2} = 7$, $x = 14$.

Solving Exponential Equations

Strategies

*Make bases equal

*Take the log of both sides

Ex 1: $4^x = \left(\frac{1}{2}\right)^{x-3}$

$(2^2)^x = (2^{-1})^{x-3}$

$2^{2x} = 2^{-x+3}$

same base

set exponents equal:

$2x = -x + 3$

$3x = 3$

$x = 1$

Ex 2: $4^x = 11$

$\log 4^x = \log 11$

$x \cdot \log 4 = \log 11$

$\frac{x \cdot \log 4}{\log 4} = \frac{\log 11}{\log 4}$

$x = 1.73$

Quick Check

1. $9^{2x} = 27^{x-1}$

$(3^2)^{2x} = (3^3)^{x-1}$

$3^{4x} = 3^{3x-3}$

$4x = 3x - 3$

$x = -3$

2. $2^x = 5$

$\log 2^x = \log 5$

$x \cdot \log 2 = \log 5$

$\frac{x \cdot \log 2}{\log 2} = \frac{\log 5}{\log 2}$

$x = 2.32$

Solving Logarithmic Equations – Solve and Check

Strategies

*Use property of equality (if same base)

* Write in exponential form

* Use properties to combine to a single log

Ex 3: $\log_5(4x-7) = \log_5(x+5)$ same base

$4x - 7 = x + 5$

$3x = 12$

$x = 4$

Check:

$\log_5(16-7) = \log_5(4+5)$

$\log_5 9 = \log_5 9$ ✓

Ex 4: $\log_4(5x-1)=3$ write in exponential

$4^3 = 5x-1$

$64 = 5x-1$

$65 = 5x$

$x = 13$

check:

$\log_4(5\cdot 13-1) = 3$

$\log_4(64)=3$ ✓

Ex 5: $\log 5x+\log(x-1)=2$ combine:

$\log 5x(x-1) = 2$

$\log(5x^2-5x) = 2$

$10^2 = 5x^2-5x$

$5x^2-5x-100 = 0$

$5(x^2-x-20) = 0$

$5(x-5)(x+4) = 0$

$x=5$ $x\neq -4$ Can't take log of a negative #

Do and Discuss

1. $\ln(3x+1)=\ln(x-5)$

$3x+1 = x-5$

$2x = 6$

$x=3$ No solution

check:

$\ln(9+1) \neq \ln(3-5)$

2. $\log(2x)+\log(x-5)=2$

$\log 2x(x-5) = 2$

$\log(2x^2-10x) = 2$

$10^2 = 2x^2-10x$

$2x^2-10x-100 = 0$

$2(x^2-5x-50) = 0$

$2(x-10)(x+5)=0$

$x=10$, $x\neq -5$

Application

Ex 6: You buy a new mountain bike for \$400. The value of the bike decreases by 15% each year. When will the bike be worth \$200?

$\frac{200}{400} = \frac{400(.85)^t}{400}$

$.5 = (.85)^t$

$\log .5 = \log .85^t$

$\frac{\log .5}{\log .85} = \frac{t\cdot \log .85}{\log .85}$

$t \approx 4.27$ years

Ex 7: At what interest rate, compounded continuously, would you need to invest \$200 in order for it to ~~double~~ triple in 6 years?

$600 = 200e^{r(6)}$

$3 = e^{6r}$

$\ln 3 = \ln e^{6r}$

$6r\cdot \ln e = \ln 3$

$\frac{6r}{6} = \frac{\ln 3}{6}$

$r = .183$

$r = 18.3\%$

Additional Resources: Textbook section 7.6 p. 515