8.5 Properties of Logarithms

- *I can use logarithm properties to expand or condense an expression*
- *I can use the change of base formula to evaluate logarithms*

Logarithm Properties		**Similar to Exponential Property**
Product Property	$\log_b m \cdot n =$ $\log_b m + \log_b n$	$a^m \cdot a^n =$ a^{m+n}
Quotient Property	$\log_b \frac{m}{n} =$ $\log_b m - \log_b n$	$\frac{a^m}{a^n} =$ a^{m-n}
Power Property	$\log_b m^n =$ $n \cdot \log_b m$	

- Write every factor in a separate log term.
- If factor is in numerator, it is a POSITIVE term.
- If factor is in denominator, it is a NEGATIVE term.
- Put exponents in front of the log.

Ex 1: Expand the logarithmic expression

a.) $\log_3 7x^2$

$\log_3 7 + \log_3 x^2$

$\log_3 7 + 2\log_3 x$

b.) $\ln x^2 y^5$

$\ln x^2 + \ln y^5$

$5\ln x + 5\ln y$

c.) $\log \frac{y^3\sqrt{z}}{4x^2}$

$\log y^3 + \log \sqrt{z} - \log 4 - \log x^2$

$3\log y + \frac{1}{2}\log z - \log 4 - 2\log x$

d.) $\log_3 \frac{7x^2}{y}$

$\log_3 7 + \log_3 x^2 - \log_3 y$

$\log_3 7 + 2\log_3 x - \log_3 y$

Quick Check:

Expand the logarithmic expression

1. $\ln \frac{y^4}{5x^6}$

$4\ln y - \ln 5 - 6\ln x$

2. $\log_5 3xy^2$

$\log_5 3 + \log_5 x + 2\log_5 y$

Ex 2: Condense the logarithmic expression.

a.) $\log_2 x + 3\log_2 y + \log_2 5$

$\log_2 x + \log_2 y^3 + \log_2 5$

$\log_2 5xy^3$

b.) $\log 2 + 3\log 3 - \log 9$

$\log 2 + \log 3^3 - \log 9$

$\log \frac{2 \cdot 3^3}{9} = \log 6$

- Bring multiples in front of log to the exponent.
- Write as ONE log with positive factors in NUMERATOR and negative factors in DENOMINTOR.

c.) $\log 2 - 3\log a - \log b$

$\log 2 - \log a^3 - \log b$

$\log \left(\frac{2}{a^3 b}\right)$

d.) $5\log 2 - 1/2 \log a - 3\log b + \log c$

$\log 2^5 - \log \sqrt{a} - \log b^3 + \log c$

$\log \left(\frac{32c}{\sqrt{a}\, b^3}\right)$

Quick Check:

Condense the logarithmic expression.

3. $\ln 3 + 2\ln x - \ln y$

$\ln \left(\frac{3x^2}{y}\right)$

4. $-4\log x - \log y + 4\log z$

$\log \left(\frac{z^4}{x^4 y}\right)$

Which of the following is equivalent to $-3\log_3 y + 5\log x - \log 3 - \log_3 (x+1)$?

$\log \frac{x^5}{3y^3(x+1)}$ $\quad$ $\log_3 \frac{1}{y^3(x+1)} + \log \frac{x^5}{3}$ $\quad$ $\log_3 \frac{1}{y^3(x+1)} \cdot \log \frac{x^5}{3}$

Change of Base Formula

$\log_b a = \frac{\log_c a}{\log_c b} = \frac{\log a}{\log b} = \frac{\ln a}{\ln b}$

Ex 3. Evaluate using common logarithms and natural logarithms

$\log_6 11$ $\quad$ $\log_5 43$ $\quad$ $\log_6 96$

$\frac{\log(11)}{\log(6)} = 1.34$ $\quad$ $\frac{\log(43)}{\log(5)} = 2.34$ $\quad$ $\frac{\log(96)}{\log(6)} = 2.55$

Ex 4. Which of the following are equivalent?

$\log_2 8$ $\quad$ $\log 2 + \log 8$ $\quad$ $3\log_2 2$ $\quad$ $\frac{\ln 8}{\ln 2}$ $\quad$ $\frac{\log 2}{\log 8}$

$\log 2 \cdot 8$ $\quad$ $\log_2 2^3$ $\quad$ $\log_2 8$ $\quad$ $\log_8 2$

Additional Resources: Section 7.5 in the Textbook