8.2: *Modeling Exponential Growth and Decay*

- In can write exponential models
- I can use exponential models

$$y = ab^x$$

any amount

initial amount

growth/ decay factor

time

Finding b: What percent (in decimal form) will you have in the future?

Rate	Growth Factor	Decay Factor
2%	102% → 1.02	98% → 0.98
7.5%	107.5% → 1.075	92.5% → 0.925
0.3%	100.3% → 1.003	99.7% → 0.997
80%	180% → 1.8	20% → 0.2
160%	260% → 2.6	not possible

Ex1: *In 2000, the population of Madison was 208,054 people. The population has been growing about 0.93% each year.*

a. Write an exponential model. 100.93% → 1.0093

$y = 208{,}054(1.0093)^x$

b. Predict the population in 2010.

$208{,}054(1.0093)^{10} \approx \boxed{228{,}233 \text{ people}}$

c. The actual population in 2010 was 228,154.
Do you think you have a good model? Yes, very close
What is the % error (difference/actual*100)?

$\frac{79}{228154} \cdot 100 = \boxed{0.035\% \text{ error}}$

d. When will you expect Madison to reach a quarter of a million people?

$Y1 = 208054(1.0093)^x$ $\boxed{x = 19.84 \text{ yrs}}$

$Y2 = 250{,}000$

x: 0 → ~~20~~ 40

y: 200,000 → 300,000

CALCULATOR:
To find an X value given Y:

[Y=] Y1 = your model
Y2 = desired value
[WINDOW]
x: 0 – (guess or look at Table)
y: 0 – past wanted or 2 #s close to wanted
[GRAPH]
[2nd] [TRACE] (CALC)
5: intersect
Scroll to intersection
Press [ENTER] 3X

Ex2: *A new television costs $1200. The value decreases about 12% each year.* 88% → 0.88

a. Write an exponential decay model giving the value of the television, y, after t years.

$y = 1200(0.88)^t$

b. Estimate the value after 2 years.

$1200(0.88)^2 = \boxed{\$929.28}$

c. When is the value of the television $300?

$Y1 = 1200(0.88)^x$

$Y2 = 300$ $\boxed{x = 10.85 \text{ yrs}}$

x: 0 → 20

y: 0 → 1200

__Quick Check:__

107% → 1.07

Ex3: *In the last 12 years a population of 38 buffalo in a park grew about 7% each year.*

a. Write an exponential growth model for the number of buffalo, b, after t years.

$b = 38(1.07)^t$

b. Estimate the population after 7 years?

$38(1.07)^7 = \boxed{61 \text{ buffalo}}$

c. After how many years was the population of about 53?

x: 0 → 10 $\boxed{x = 4.92 \text{ yrs}}$

y: 30 → 60

Half – Life: The amount of time it takes a radioactive substance to decay by half

- Because it is a Half-life, the decay factor will always be ½ or 0.5
- We want the exponent to represent the number of half-lives, so it will be x/half-life
- A common use of half-lives is for radio carbon dating. The half life of C-14 is 5730 years

Half-Life Model:

$$y = a(0.5)^{x/\text{half-life}}$$

Ex4: Sodium-24 has a half-life of 15 hours. An 18.0g sample is being used.

a. Write an exponential model for Sodium-24.

$$y = 18(.5)^{x/15}$$

b. How much sodium-24 will remain after 60 hours? After 3 days?

$18(.5)^{(60/15)} = \boxed{1.125g}$ $\quad$ $18(.5)^{(72/15)} = \boxed{0.65g}$

c. When will approximately 0.25 grams remain? (Calculator)

x: 0 → 100
y: 0 → 1

$\boxed{x = 92.55 \text{ hours}}$ or 3 days 20.55 hrs

Ex5: After 42 days, a 2.0g sample of phosphorus-32 contains only 0.25g of isotope. What is the half-life of phosphorus-32?

$0.25 = 2(.5)^{42/h}$ $\quad$ $Y1 = 2(.5)^{42/x}$ $\quad$ $Y2 = 0.25$

x: 0 → 40
y: 0 → 1

$\boxed{X = 14 \text{ days}}$

Alternative:
$Y1 = 2(.5)^x$
$Y2 = 0.25$
x = 3 half lives
42/3 = 14 days

Ex6: In 2016, you find a very old painting in an attic. A piece of the canvas is carbon dated. The lab finds 3.8g of the original 4g of C-14 remain. When is it believed the painting was made?

$3.8 = 4(.5)^{x/5730}$ $\quad$ $Y1 = 4(.5)^{x/5730}$ $\quad$ $Y2 = 3.8$

x: 0 → 5000
y: 3 → 4

x = 424.02 years
2016 − 424
$\boxed{\text{In 1592 AD}}$

Do & Discuss

Ex7: Suppose an animal carcass originally contains 25g of C-14.

a. Write an exponential model

$$y = 25(.5)^{x/5730}$$

b. How much C-14 remains after ten thousand years?

$$25(.5)^{\frac{10000}{5730}} = \boxed{7.46\text{ g}}$$

c. When will 1 gram remain?

$Y1 = 25(.5)^{x/5730}$
$Y2 = 1$

x: 0 →
y: 0 → 2

$\boxed{x = 26{,}609.3 \text{ years}}$

Estimate ½ lives: 25 → 12.5 → 6.25 → ≈3 → 1.5 → .75
1 2 3 4 5 × 5730 = 28650 yrs