7.3 Series

- I can write a series using sigma notation (summation notation)
- I can find sums of finite arithmetic and geometric series

VOCABULARY

Sequence - an ordered list of numbers **Ex:** 1, 3, 5, 7

Series – the sum of the terms of a sequence. **Ex:** 1 + 3 + 5 + 7

A series can be finite (ending) or infinite (goes on forever).

$\sum$ - This is the capital Greek letter called Sigma. It means Sum.

Sigma Notation (also called "summation notation")

(where to end)
last value of n

formula for the terms

$$\sum_{n=1}^{6} 4n = 4(1) + 4(2) + 4(3) + 4(4) + 4(5) + 4(6) = 4 + 8 + 12 + 16 + 20 + 24 = 84$$

Index of summation

first value of n
(where to start)

Ex 1: Write out the series and find the sum

a. $\sum_{i=2}^{5} i^2 - 1$

$(2^2-1) + (3^2-1) + (4^2-1) + (5^2-1)$

$3 + 8 + 15 + 24 = 50$

b. $\sum_{j=0}^{4} 5 - 2j$

$(5-2\cdot 0) + (5-2\cdot 1) + (5-2\cdot 2) + (5-2\cdot 3) + (5-2\cdot 4)$

$5 + 3 + 1 + -1 + -3$

$= 5$

Ex 2: Write the series using summation notation. (Hint: Is the series arithmetic, geometric, or neither?)

Arith
$d=2$

a. 5 + 7 + 9 + 11 + 13 + 15

Find formula:

$a_n = 5 + 2(n-1) = 3 + 2n$

$$\boxed{\sum_{n=1}^{6} 3 + 2n}$$

Geom
$r = -2$

b. -5 + 10 + -20 + 40 + -80 + . . .

Find formula:

$a_n = -5(-2)^{n-1}$

$$\boxed{\sum_{n=1}^{\infty} -5(-2)^{n-1}}$$

Quick Check

1. Write out the series and find the sum

$$\sum_{k=5}^{9} k^2 + 3$$

$(5^2+3)+(6^2+3)+(7^2+3)+(8^2+3)+(9^2+3)$

$28 + 39 + 52 + 67 + 84$

$= 270$

2. Write the series using summation notation

16 + 10 + 4 + -2 + -8

Arith. $d = -6$

$a_n = 16 + -6(n-1) = 22 - 6n$

$$\boxed{\sum_{n=1}^{5} 22 - 6n}$$

Sums of Series

Sum of Finite Arithmetic Series	Sum of Finite Geometric Series
$S_n = n\left(\frac{a_1 + a_n}{2}\right)$ where S_n is the sum of n terms a_1 is the first term a_n is the nth term.	$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$ where S_n is the sum of n terms a_1 is the first term r is the common ratio

Ex 3: Determine whether the series is arithmetic or geometric and then find its sum.

a. $\sum_{k=1}^{16} 4(3)^{k-1} = 4(3)^0 + 4(3)^1 + 4(3)^2 + \ldots$

$4 + 12 + 36 + \ldots$

Geom $r = 3$
(Can also see that from formula)

$S_{16} = 4\left(\frac{1-3^{16}}{1-3}\right)$

$= \boxed{86,093,440}$

b. $\sum_{j=1}^{22} -9 + 11j$

Arith $d = 11$

$a_1 = -9 + 11(1) = 2$

$a_{22} = -9 + 11(22) = 233$

$S_{22} = 22\left(\frac{2+233}{2}\right)$

$\boxed{S_{22} = 2585}$

c. 2 + 6 + 10 + 14 + . . . +58

what term # is this?

Arith
$d = 4$

$a_1 = 2$
$a_{15} = 58$

$58 = 2 + (n-1)\cdot 4$
$58 = 2 + 4n - 4$
$60 = 4n$
$n = 15$

$S_{15} = 15\left(\frac{2+58}{2}\right)$

$\boxed{S_{15} = 450}$

d. A regional soccer tournament has 64 teams. In the first round, 32 games are played. In each round, the number of teams decreases by ½. Write a rule for the number of games played in the nth round. Find the total number of games played in the tournament.

Geom $r = 1/2$ There will be 6 rounds

$a_n = 32 \cdot \left(\frac{1}{2}\right)^{n-1}$

$S_6 = 32\left(\frac{1-\left(\frac{1}{2}\right)^6}{1-\frac{1}{2}}\right) = \boxed{63 \text{ games}}$

- *Additional Resources:* Textbook Chapter 12.2 pg. 802 and 12.3 pg.810