7.2 Recursive Rules for Sequences

- I can list the first few terms of a sequence given the recursive rule
- I can write recursive rules for sequences

Vocabulary

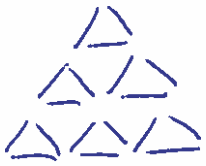
Recursive Rule:
1. Gives the first term(s) of the sequence
2. Gives an equation to find a_n using the previous term(s)

Notation: 1st term = a_1 2nd term = a_2 nth term = a_n

Previous term = a_{n-1} 2 terms prior = a_{n-2}

What's the point? Recursive rules are often used in spreadsheets to fill columns of data quickly and to create new lists of data from previous lists.

Toothpicks:



How many toothpicks are in the 1st row? 3

How many toothpicks are in the 2nd row? 6

How many toothpicks are in the 3rd row? 9

What is the pattern? add 3

Recursive Rule: $a_1 = 3$

$a_n = a_{n-1} + 3$

Ex. 1: Give the first 5 terms of the sequence.

a. $a_1 = 3$, $a_n = a_{n-1} - 6$

3, -3, -9, -15, -21

b. $a_0 = 1$, $a_n = -5a_{n-1}$

1, -5, 25, -125, 625

c. $a_0 = 1$, $a_1 = 2$, $a_n = a_{n-1} \cdot a_{n-2}$

1, 2, 2, 4, 8

Ex2: Write a recursive rule for the sequences.

a. 512, 128, 32, 8,... (×1/4 ×1/4 ×1/4)

$a_1 = 512$

$a_n = \frac{1}{4}a_{n-1}$ or $\frac{a_{n-1}}{4}$

b. 2, 4, 16, 256,... (2^1 4^2 16^2)

$a_1 = 2$

$a_n = (a_{n-1})^2$

c. x, -x, -3x, -5x,... (-2x -2x -2x)

$a_1 = x$

$a_n = a_{n-1} - 2x$

Quick Check: Write recursive rules for #1-8 on your INTRO to Sequences worksheet

Some sequences require a little more creativity to figure out the pattern.

You need to know the **pattern between terms** to be able to write the recursive rule.

Things to ask...

1. Is more than one operation happening to the previous term?
   - Is the previous term doubling/squaring/halving?
   - Then is it adding/subtracting an amount?
2. Is more than one previous term involved?
   - The previous 2 terms?
   - The previous term & 2 before that one?
3. Is the previous term and the term number (n) being used together?

Fibonacci Sequence: 1, 1, 2, 3, 5, 8,... add 2 previous terms → need first 2 terms

$f_1 = 1 \quad f_2 = 1$

$f_n = f_{n-1} + f_{n-2}$

Ex 3: Write a recursive rule for the sequences.

a. $7, 14, 42, 168, 840, \ldots$

$a_1 = 7$

$a_n = n \cdot a_{n-1}$

b. 14, 20, 6, -14, -20,...

20-14 6-20 -14-6

$a_1 = 14 \quad a_2 = 20$

$a_n = a_{n-1} - a_{n-2}$

c. -5. 2. -10, -20, 200,...

$a_1 = -5 \quad a_2 = 2$

$a_n = a_{n-1} \cdot a_{n-2}$

d. 1, 4, 9, 16, 25,...

+3 +5 +7 +9

$a_1 = 1$

$a_n = a_{n-1} + 2n - 1$ or $\left(\sqrt{a_{n-1}} + 1\right)^2$

e. 1, 2, 4, 7, 11,...

$a_1 = 1$

$a_n = a_{n-1} + n - 1$

f. 8, 27, 64, 125, 216,...

$2^3, 3^3, 4^3$

$a_1 = 8$

$a_n = \left(\sqrt[3]{a_{n-1}} + 1\right)^3$

Quick Check: On the Intro to Sequences worksheet write recursive rules for #10, 12, 13, 16, 18

Additional Resources:

- Textbook 12.5 pg.827
- www.regentsprep.org/regents/math/algtrig/atp3/recursive.htm