7.1 Explicit Rules for Sequences

- I can identify arithmetic and geometric sequences.
- I can write explicit rules for sequences.

Vocabulary

A Sequence is a string of objects, like numbers, that follow a particular pattern. The individual elements in a sequence are called terms.

$a_1\ a_2\ a_3\ a_4$

Finite sequence: 2, 4, 6, 8

$a_1\ a_2\ a_3\ a_4 \ldots a_n$

Infinite Sequence: 2, 4, 6, 8, …

a_1 is the 1st term

a_4 is the 4th term

a_n is the nth term.

An ARITHMETIC sequence has a common difference so the same value is added

An GEOMETRIC sequence has a common ratio so the same value is multiplied

Ex. 1: Decide whether the sequence is arithmetic, geometric, or neither.

a. 512, 128, 64, 8,…

Geometric

b. 32, 27, 21, 14, 6, . . .

Neither

c. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \ldots$

neither

Quick Check

Decide if the sequences on the Intro to Sequences worksheet are arithmetic, geometric, or neither. Circle the correct response.

EXPLICIT RULES FOR SEQUENCES

An explicit rule is a function to describe any term based on its position in the pattern.

1st term

Arithmetic Explicit Rule: $a_n = a_1 + (n-1)d$

difference

Geometric Explicit Rule: $a_n = a_1 r^{n-1}$

1st term ratio

Neither: relate the term number (n) to the term (a_n)

Ex 2: Write a rule for the nth term of the sequence.

a. 32, 27, 22, 17, 12, . . .

arithmetic $d=-5$ $a_1=32$

$a_n = 32+(n-1)(-5)$

$\boxed{a_n = -5n+37}$

b. −15, −9, −3, 3, . . .

arithmetic $d=6$ $a_1=-15$

$\boxed{a_n = 6n-21}$

c. 512, 128, 64, 8,…

geometric $r=\frac{128}{512}=\frac{1}{4}$

$\boxed{a_n = 512\left(\frac{1}{4}\right)^{n-1}}$

d. 14, 28, 56, 112,…

geometric $r=\frac{28}{14}=\frac{56}{28}=\frac{112}{56}=2$

$\boxed{a_n = 14(2)^{n-1}}$

e. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$

term 1 2 3 4 n

neither $\boxed{a_n = \frac{1}{n+1}}$

f. 8, 27, 64, 125, 216, …

2^3 3^3 4^3 5^3 6^3

term 1 2 3 4 5

neither. $\boxed{a_n = (n+1)^3}$

Quick Check

On the Intro to Sequences worksheet write explicit rules for #1-5, 13,14

Ex 3: One term of an arithmetic sequence is $a_{11} = 41$. The common difference is $d = 5$.

$a_n = a_1 + (n-1)d$

$a_n = -9 + (n-1)(5)$

$41 = a_1 + (11-1)(5)$

$41 = a_1 + 50$

$-9 = a_1$

$\boxed{a_n = 5n-14}$

Ex 4: One term of a geometric sequence is $a_3 = -18$. The common ratio is $r = 3$. Write a rule for the nth term.

$a_n = a_1 r^{n-1}$

$-18 = a_1 (3)^{3-1}$

$-18 = a_1(9)$

$-2 = a_1$

$\boxed{a_n = -2(3)^{n-1}}$

Additional Resources: In the textbook Sections 12.1-12.2 and www.khanacademy.org/math/algebra/sequences