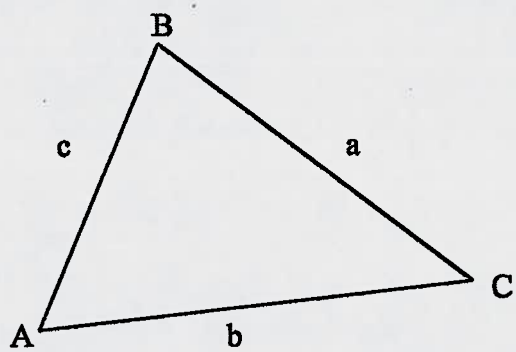
6.2 Law of Sines & Law of Cosines

- I can find solve a non-right triangle
- I can set up and solve a triangle in an application



Law of Sines:

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

Used for triangles cases of:

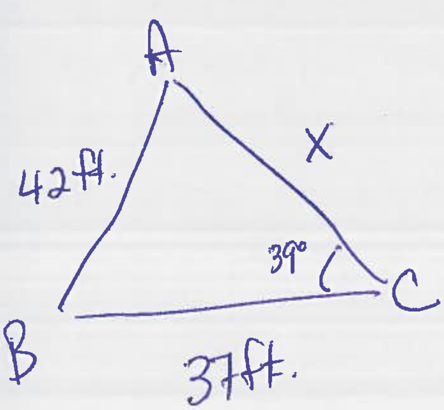
Law of Cosines:

$$a^2=b^2+c^2-2bc\cos A$$
$$b^2=a^2+c^2-2ac\cos B$$
$$c^2=a^2+b^2-2ab\cos C$$

Used for triangles cases of:

Using Law of Sines

Ex 1: You are seeding a triangular courtyard. One side of the courtyard is 42 feet long and another side is 37 feet long. The angle opposite the 42 foot side is 39°. Find the length of the third side of the garden.



$$\frac{\sin 39°}{42}=\frac{\sin A}{37}$$

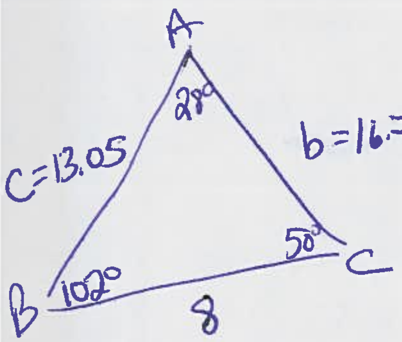
$A=33.7°$

$180-33.7-39=107.3°=B$

$$\frac{\sin 107.3}{x}=\frac{\sin 39°}{42}$$

$$\boxed{x=63.7\text{ ft}}$$

Ex 2: Solve the triangle ABC with A=28°, B=102°, and a=8.



$$\frac{\sin 28^\circ}{8} = \frac{\sin 102^\circ}{b}$$

$$\boxed{b = 16.7}$$

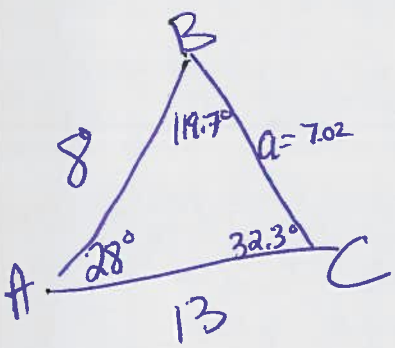
$$180 - 28 - 102 = \boxed{50^\circ = C}$$

$$\frac{\sin 50^\circ}{c} = \frac{\sin 28^\circ}{8}$$

$$\boxed{c = 13.05}$$

Using Law of Cosines

Ex 3: Solve the triangle ABC with A=28°, b=13, and c=8.



$$a^2 = 8^2 + 13^2 - 2(8)(13)\cos 28^\circ$$

$$\sqrt{a^2} = \sqrt{49.35}$$

$$\boxed{a = 7.02}$$

Then use law of sines to find angles less than 90°. B will be the largest since side b is the largest. So find angle C

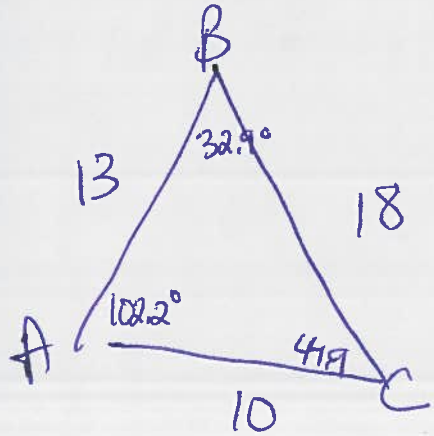
$$\frac{\sin C}{8} = \frac{\sin 28^\circ}{7.02}$$

$$\boxed{C = 32.3^\circ}$$

$$180 - 32.3 - 28 =$$

$$\boxed{B = 119.7^\circ}$$

Ex 4: Solve the triangle ABC with a=18, b=10, and c=13.



$$18^2 = 13^2 + 10^2 - 2(13)10\cos A$$

$$324 = 269 - 260\cos A$$

$$\frac{55}{-260} = \frac{-260\cos A}{-260}$$

$$-.2115... = \cos A$$

$$\cos^{-1}(-.2115...)$$

$$\boxed{A = 102.2^\circ}$$

$$\frac{\sin 102.2}{18} = \frac{\sin C}{13}$$

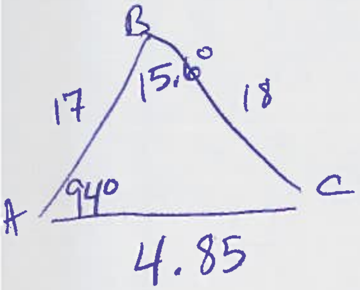
$$\boxed{C = 44.9^\circ}$$

$$180^\circ - 44.9^\circ - 102.2^\circ$$

$$\boxed{B = 32.9^\circ}$$

Quick Check:

1. Solve the triangle ABC with A=94°, a=18, and c=17.



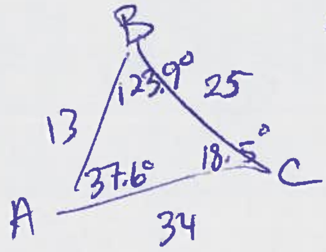
$\frac{\sin 94°}{18} = \frac{\sin C}{17}$

$C = 70.4°$

$\frac{\sin 15.6}{b} = \frac{\sin 94}{18}$

$b = 4.85$

2. Solve the triangle ABC with a=25, b=34, and c=13.



$25^2 = 13^2 + 34^2 - 2(13)(34)\cos A$

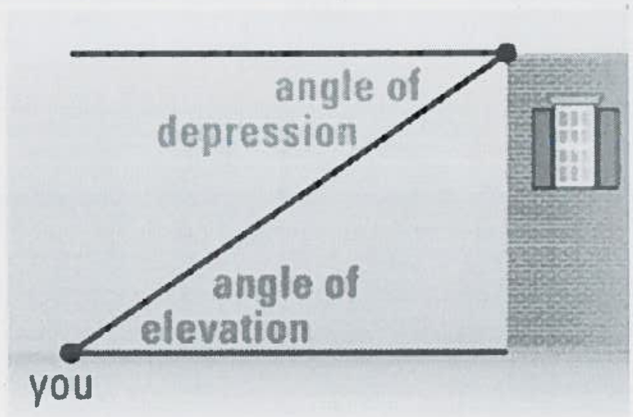
$A = 37.6°$

$\frac{\sin 37.6°}{25} = \frac{\sin C}{13}$

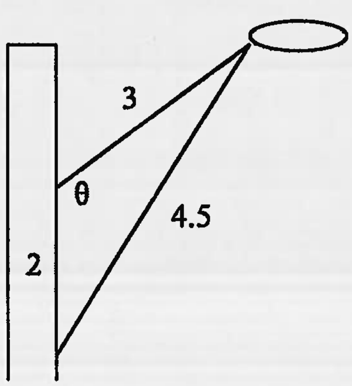
$18.5°$

$B = 123.9$

Applications



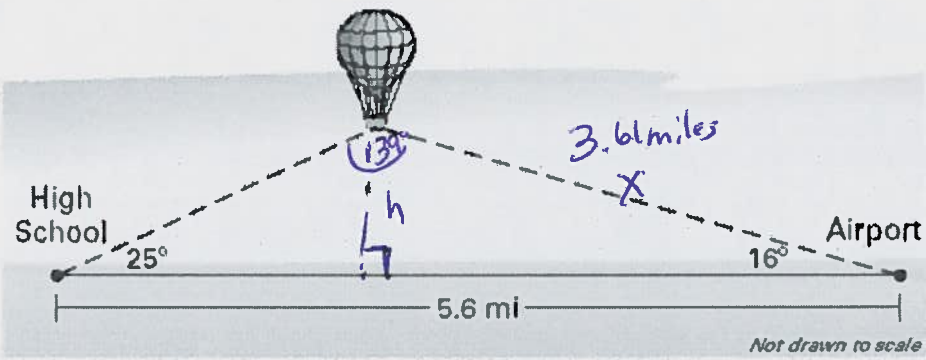
A. Determine the angle θ in the design of the street light shown.



$4.5^2 = 2^2 + 3^2 - 2(2)(3)\cos\theta$

$\boxed{\theta = 127.2°}$

B. A hot air balloon is floating between the high school and the airport which are 5.6 miles apart. The angle of elevation from the high school is 25° and the angle of elevation from the airport is 16°. Approximate the height of the balloon above the ground.



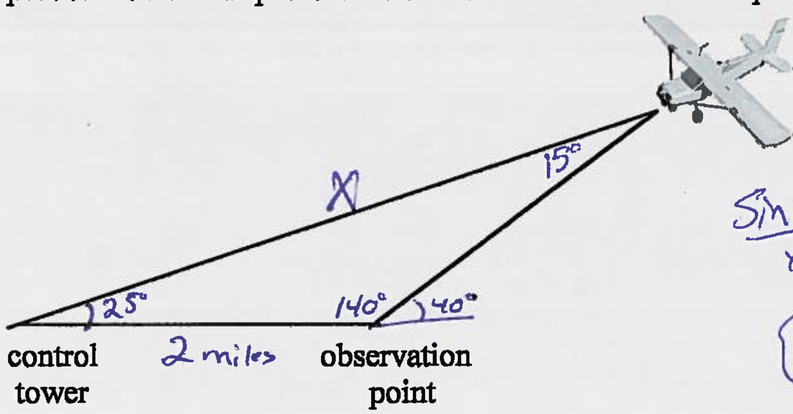
$\frac{\sin 139^\circ}{5.6} = \frac{\sin 25}{x}$

$x = 3.61$mi

$\sin 16^\circ = \frac{h}{3.61}$

$\boxed{h = .995 \text{ miles}}$

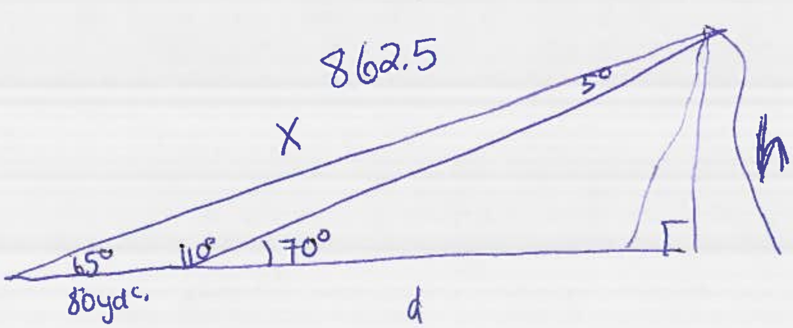
C. An airport control tower and an observation post are 2 miles apart. The angle of elevation from the airport control tower to an airplane is 25°. The angle of elevation from the observation post to the same airplane is 40°. Find the distance that the plane is from the control tower.



$\frac{\sin 140^\circ}{x} = \frac{\sin 15^\circ}{2}$

$\boxed{x = 4.97 \text{ miles}}$

D. You are on level ground at an unknown distance *d* (in yards) from the base of a mountain that is *h* yards tall. The angle of elevation to the top of the mountain is 70°. You step back 80 yards and measure the angle of elevation to be 65°. Find the height *h* of the mountain.



$\frac{\sin 110^\circ}{x} = \frac{\sin 5^\circ}{80}$

$x = 862.5$yds.

$\sin 65^\circ = \frac{h}{862.5}$

$\boxed{h = 781.7 \text{ yds}}$