6.1 Right Triangle Trigonometry

Key

- I can identify and use the 6 trigonometric ratios
- I can solve a right triangle

VOCABULARY

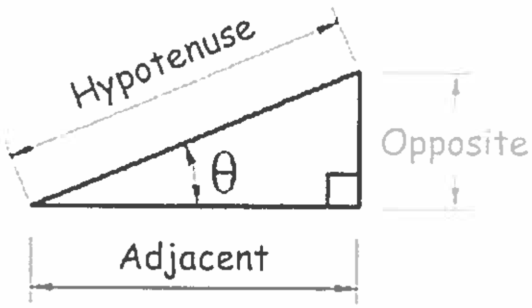
Trigonometry – From Greek "Trigon" meaning triangle and "metron" meaning measure

Trigonometric Ratio – a ratio of side lengths in a right triangle.

Solving a Right Triangle – finding all the unknown side lengths and angle measures.

Pythagorean Theorem - $a^2+b^2=c^2$ where a + b are legs + c is hypotenuse

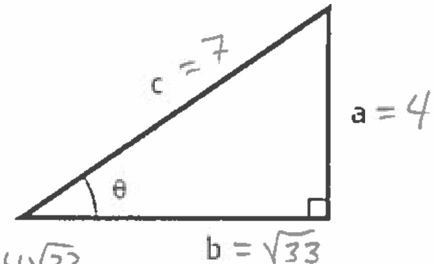
There are 6 trigonometric ratios. We learned **sine, cosine and tangent** in Geometry. Remember **SohCahToa**?



	Opposite		Cosine		Hypotenuse		Opposite	
S	O	H	C	A	H	T	O	A
Sine		Hypotenuse		Adjacent		Tangent		Adjacent

The 3 other ratios are **cosecant, secant and cotangent**. They are reciprocals of the original three

$\sin\theta = \frac{opp}{hyp}$	$\csc\theta = \frac{1}{\sin\theta}$	$\csc\theta = \frac{hyp}{opp}$
$\cos\theta = \frac{adj}{hyp}$	$\sec\theta = \frac{1}{\cos\theta}$	$\sec\theta = \frac{hyp}{adj}$
$\tan\theta = \frac{opp}{adj}$	$\cot\theta = \frac{1}{\tan\theta}$	$\cot\theta = \frac{adj}{opp}$



Ex 1: Evaluate the 6 trigonometric ratios of the angle θ if a = 4 and c = 7.

sin θ= $\frac{4}{7}$ cos θ= $\frac{\sqrt{33}}{7}$ tan θ= $\frac{4}{\sqrt{33}} = \frac{4\sqrt{33}}{33}$

~~cos~~ csc θ= $7/4$ sec θ= $\frac{7}{\sqrt{33}} = \frac{7\sqrt{33}}{33}$ cot θ= $\frac{\sqrt{33}}{4}$

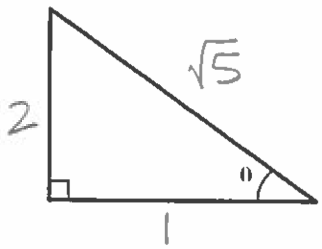
Pythag:

$4^2+b^2=7^2$

$b=\sqrt{33}$

Ex 2: Let θ be an acute angle in a right triangle. Find the value of the other 5 trig ratios of θ if sec θ = $\sqrt{5}$.

$\cos\theta = \frac{1}{\sqrt{5}}$



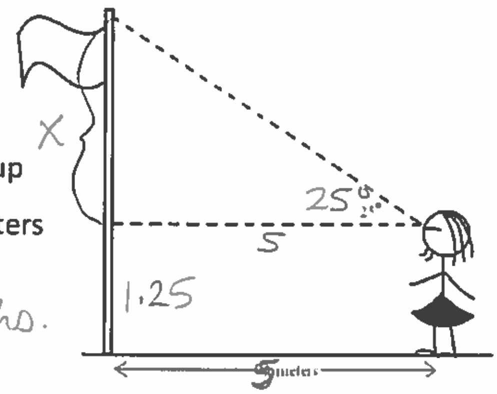
sin θ= $\frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ cos θ= $\frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ tan θ= $2/1 = 2$

~~cos~~ csc θ= $\frac{\sqrt{5}}{2}$ sec θ= $\frac{\sqrt{5}}{1} = \sqrt{5}$ cot θ= $1/2$

Pythag: $1^2 + b^2 = (\sqrt{5})^2$

$1 + b^2 = 5$ $b = 2$

Using trig ratios to find missing side lengths:



Ex 3: Rachel is standing 5 meters from the base of a flagpole. She looks up to the top of the flagpole at an angle of 25°. If Rachel's eyes are 1.25 meters above the ground, how high is the top of the flagpole? Round to hundreths.

* use tan

$\tan 25^\circ = \frac{x}{5}$

$x \approx 2.33$

$2.33 + 1.25 = 3.58\,m$

$x = 5 \cdot \tan 25^\circ$

$x = 5 \cdot (.4663)$

Inverse trig ratios:

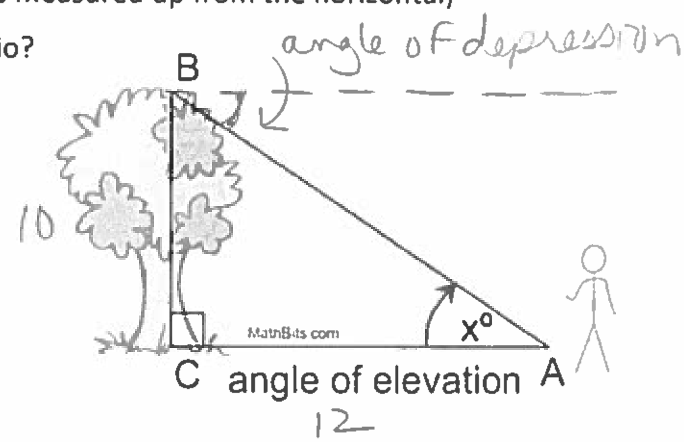
Use an **inverse ratio** when you want to know the angle that has a given trig ratio.

Notation: $\sin^{-1}(.5)$ means what angle has a sine ratio of .5?

On your calculator: Press 2nd sin (.5) What angle has a sine of .5? 30°

Ex 4: You are standing 12 feet from the base of a tree that is 10 feet tall. At what angle are you looking up at the tree? (This is called the "angle of elevation" – the angle measured up from the horizontal)

*Hint – you are looking for the angle that has what trig ratio?



* use tangent

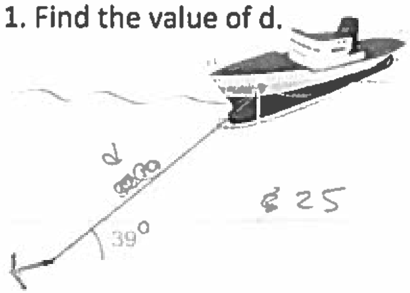
$\tan x = \frac{10}{12}$

* what angle has a tan of $\frac{10}{12}$?

$\tan^{-1}(10/12) \approx 39.8^\circ$

Quick Check

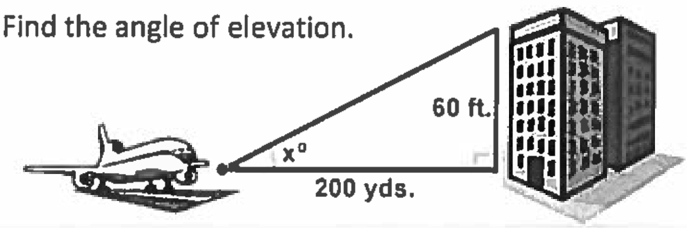
1. Find the value of d.



$\frac{\sin 39^\circ}{1} = \frac{25}{d}$ $\quad d = \frac{25}{\sin 39^\circ}$

$d \cdot \sin 39 = 25$ $\quad d \approx 59.16\,m$

2. Find the angle of elevation.



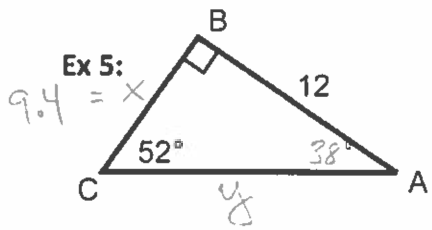
$\tan x = \frac{60}{200} = \frac{6}{20} = \frac{3}{10}$

$\tan(3/10) = 16.7^\circ$

Solving Right Triangles

Use the Pythagorean Theorem, trig and inverse trig to find all missing sides and angle measures.

Ex 5:



$m\angle A =$ 38° **CB =** 9.4 **CA =** 15.2

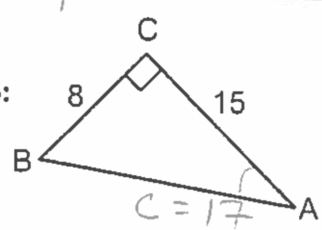
$\frac{\tan 52^\circ}{1} = \frac{12}{x}$

$x = \frac{12}{\tan 52^\circ}$ $\quad x = 9.4$

$\frac{\sin 52^\circ}{1} = \frac{12}{y}$

$y = \frac{12}{\sin 52^\circ}$ $\quad y = 15.2$

Ex 6:



BA = 17 $m\angle A =$ 28.1° $m\angle B =$ 61.9°

Pythag: $8^2 + 15^2 = c^2$

$c = 17$

$\tan A = \frac{8}{15}$

$\tan^{-1}(8/15) \approx 28.1^\circ$

- *Additional Resources:* Textbook Chapter 13.1 pg. 852