Key

5.2 Finding and Using Roots to Sketch Graphs of Polynomial Functions

- I can identify the number of roots of a polynomial
- I can find the roots of a polynomial by factoring
- I can use the roots to sketch a graph of the polynomial function without a graphing calculator

KEY IDEAS

*A polynomial of degree __n__ has exactly __n__ solutions.
(Repeated solutions must be counted separately).

* The solutions could be __real__ and/or __imaginary__.

*Real solutions correspond to the __x-intercepts__ of the graph.

*A polynomial with an __odd__ degree always has at least one __real solution (x-int)__

*If (x – k) is a factor of the polynomial, then __x = k__ is a solution of the polynomial equation.

Ex 1: Determine the number of solutions of the polynomial (Hint: What is the degree?)

a). $y = 2 - 3x^2 + 8x^5$

5

b). $y = -2(x-6)^2 + 7$

2

c). $y = 3x(x-4)(x+8)^2$

4

Ex 2: For the factored polynomials, a)state the degree, b)find the solutions, and c)state the # of x-intercepts

a). $y = -6(x-7)(2x+1)$

a) __2__

b) __$x = 7, -1/2$__

c) __2__

b). $y = 4x(x-9)^2(x+2)$

a) __4__

b) __$x = 0, 9, 9, -2$__

c) __4__
(9 is a double root)

c). $y = 9x^2(3x+5)(x^2+4)$

a) __5__

b) __$x = 0, 0, -5/3, 2i, -2i$__

c) __3__
(zero is a double)

Ex 3: a) State the degree and b) Find the solutions of the polynomials by factoring. (Look for a GCF first).

a). $y = 2x^3 - 8x^2 - 24x$

Degree: 3 GCF: 2x

$2x(x^2 - 4x - 12) = 0$

$2x(x-6)(x+2) = 0$

$\boxed{x = 0, 6, -2}$

b). $y = 6x^4 - 21x^3 - 12x^2$

Degree: 4 GCF: $3x^2$

$3x^2(2x^2 - 7x - 4) = 0$

	2x	1
x	x	-4

$-8x$

$3x^2(2x+1)(x-4) = 0$

$\boxed{x = 0, 0, -\frac{1}{2}, 4}$

Quick Check

State the degree of the polynomial and then find the solutions.

1. $y = -4x(3x-2)(x^2+5)$

Degree: 4

$\boxed{x = 0, 2/3, \pm i\sqrt{5}}$

$x^2 = -5 \quad x = \pm i\sqrt{5}$

2. $y = 6x^3 - 21x^2 - 45x$

Degree: 3 GCF: 3x

2x	3
x	-5

$3x(2x^2 - 7x - 15) = 0$

$3x(2x+3)(x-5) = 0$

$\boxed{x = 0, -3/2, 5}$

Sketching Graphs of Polynomial Functions (without a graphing calculator)

Recall:

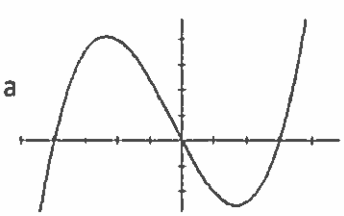
Degree and leading coefficient determine the end behavior and shape of the graph.

The real solutions correspond to the x-intercepts of the graph.

Ex 4: Determine which could be the graph of $y = -2x(x-3)(x+4)$. Explain your reasoning for each.

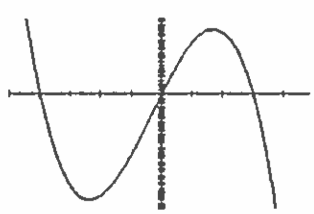
Hint- First find: Degree 3, LC -2 and Solutions $x = 0, 3, -4$

a



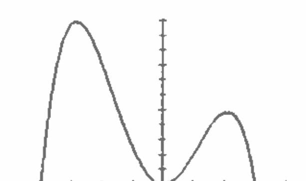
No - LC is neg

b.



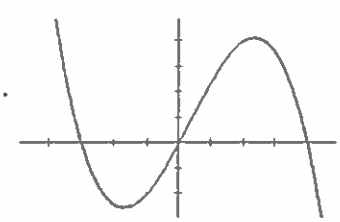
yes
- correct x-int
- LC neg
- odd deg. shape

c.



No
- Even degree shape

d.



No - wrong x-intercepts

Ex 5: $y=-7(x-1)(x+3)^2$

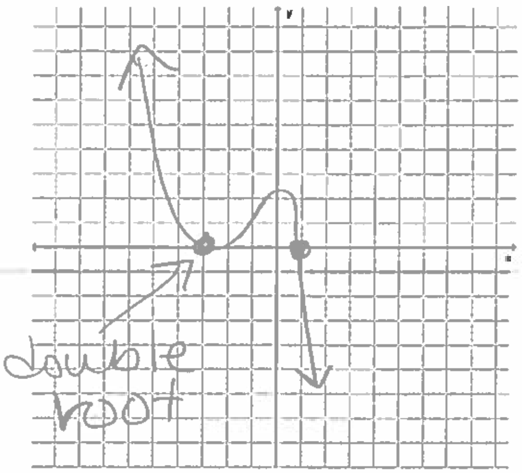
a). What is the degree? 3 b). LC −7

c). Basic shape of graph________ d). How many solutions? 3

e). Find solutions x = 1, −3, −3

f). How many x-intercepts? 3 g). Sketch ~~the~~ a graph

(−3 double)



Do and Discuss

1. $y=4(x-2)(x+3)(x+8)(x^2+4)$

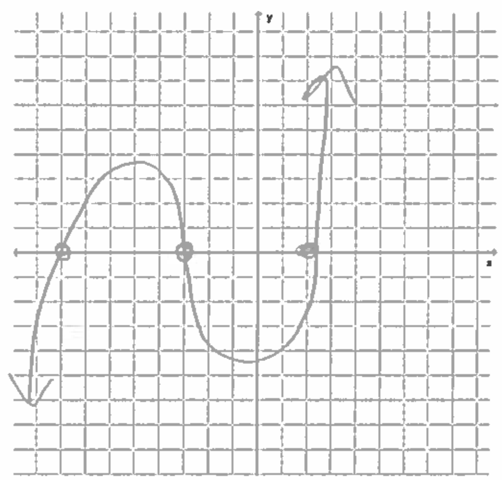
a). What is the degree? 5 b). LC 4

c). Basic shape of graph________ d). How many solutions? 5

e). Find solutions x = 2, −3, −8, 2i, −2i

f). How many x-intercepts? 3 g). Sketch ~~the~~ a graph

(real solutions)



2. $y=8x(x-3)(x+5)^2$

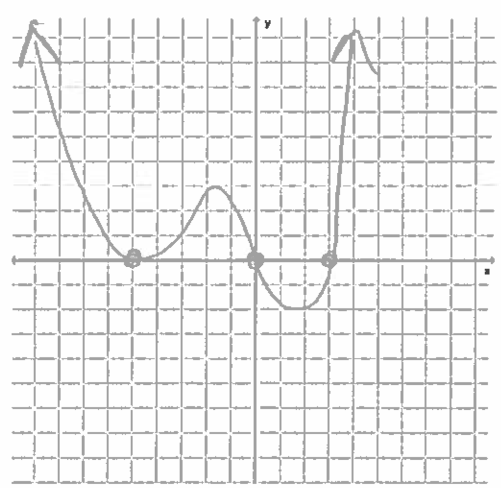
a). What is the degree? 4 b). LC 8

c). Basic shape of graph________ d). How many solutions? 4

e). Find solutions x = 0, 3, −5, −5

f). How many x-intercepts? 4 g). Sketch ~~the~~ a graph

(−5 is double)



- *Additional Resources:* Textbook Chapter 5.4 pg. 353 and 5.7 pg.379