

(Key)

4.4 Solving Equations with Exponents and Radicals

- I can solve an equation with exponents - including whole number and fractional exponents
- I solve equations with radicals
- I can check for extraneous solutions

Taking the **even** root of a number, "a" (square root, 4th root, 6th root, etc):

If $a > 0$, <u>2</u> real nth roots.	Ex: 4 th roots of 16 are <u>2</u> and <u>-2</u> Since $2^4 = 16$ and $(-2)^4 = 16$
If $a = 0$, <u>1</u> real nth roots.	Ex: $\sqrt{0} = 0$ Since $0^2 = 0$
If $a < 0$, <u>0</u> real nth roots.	Ex: $\sqrt{-36}$ has no real solution (we know it has imaginary solutions, though)

Taking the **odd** root of a number, "a" (cube root, 5th root, 7th root, etc):

If $a > 0$, <u>1</u> real nth root.	Ex: $\sqrt[5]{32} = 2$ Since $2^5 = 32$
If $a = 0$, <u>1</u> real nth root.	Ex: $\sqrt[7]{0} = 0$ Since $0^7 = 0$
If $a < 0$, <u>1</u> real nth root.	Ex: $\sqrt[3]{-27} = -3$ Since $(-3)^3 = -27$

Solving Equations with Exponents:

Ex 1: $4x^5 - 2 = -130$

$$\frac{4x^5}{4} = \frac{-128}{4}$$

$$x^5 = -32$$

$$x = \sqrt[5]{-32}$$

take the 5th root
(1 solution)

$x = -2$

Ex 2: $3(x+2)^4 + 5 = 101$

$$\frac{3(x+2)^4}{3} = \frac{96}{3}$$

$$(x+2)^4 = 32$$

* take 4th root (2 solns)

$$x+2 = \pm \sqrt[4]{32}$$

$$x = -2 \pm \sqrt[4]{16} \cdot \sqrt[4]{2}$$

$$x = -2 \pm 2\sqrt[4]{2}$$

Ex 3: $2(x+4)^{\frac{2}{3}} + 1 = 33$

$$\frac{2(x+4)^{\frac{2}{3}}}{2} = \frac{32}{2}$$

$$(x+4)^{\frac{2}{3}} = 16$$

$$x+4 = \pm 16^{\frac{3}{2}}$$

$$x+4 = (\pm \sqrt{16})^3$$

$$x+4 = \pm 64$$

* take reciprocal power
(2 solutions)

$x = 60$
or $x = -68$

Steps:

1. Get the quantity raised to the power alone
2. Undo the power (with a root or reciprocal power)
3. Solve (might have 0, 1, or 2 solutions)
4. Check for extraneous solutions

Quick Check

1. $5(x+2)^3 + 10 = 130$

2. $6x^{\frac{4}{3}} - 8 = 88$

$$x = -2 \pm 2\sqrt[3]{3}$$

$$x = \pm (\sqrt[4]{16})^3$$

$$x = \pm 8$$

Solving Equations with Radicals:

Ex 4: $1 + \sqrt{x+6} = 4$

$$\begin{array}{r} -1 \qquad -1 \\ \hline (\sqrt{x+6})^2 = 3^2 \end{array}$$

$$x+6 = 9$$

$$x = 3$$

Check: $1 + \sqrt{3+6} \stackrel{?}{=} 4$
 $4 = 4 \checkmark$

Ex 5: $(x-2)^2 = (\sqrt{x+10})^2$

$$(x-2)(x-2) = x+10$$

$$x^2 - 4x + 4 = x + 10$$

$$x^2 - 5x - 6 = 0$$

$$(x-6)(x+1) = 0$$

$$x = 6 \text{ or } x = -1$$

Check: $x = 6 : 4 \stackrel{?}{=} \sqrt{6+10}$
 $4 = 4 \checkmark$

$x = -1 : -3 \stackrel{?}{=} \sqrt{-1+10}$
 $-3 \neq 3$ Extraneous solution.

$$x = 6$$

Steps:

1. Get the root alone
2. Undo the root (with a power)
3. Solve (might have 0, 1, or 2 solutions)
4. Check for extraneous solutions

Do and Discuss

In a hurricane, the mean sustained wind velocity (v) in meters per second is given by the function $v(p) = 6.3\sqrt{1013-p}$ where p is the air pressure, in millibars, at the center of the hurricane. Estimate the air pressure at the center of the hurricane when the mean sustained wind velocity is 54.5 m/sec. Round your answer to nearest hundredth.

$$\frac{54.5}{6.3} = \frac{6.3\sqrt{1013-p}}{6.3}$$

$$8.65^2 = (\sqrt{1013-p})^2$$

$$\begin{array}{r} 74.84 = 1013 - p \\ -1013 \quad -1013 \end{array}$$

$$-938.16 = -p$$

$$p = 938.16$$

Check: $54.5 \stackrel{?}{=} 6.3\sqrt{1013-938.16}$
 $54.5 = 54.5 \checkmark$