4.3 Rational Exponents

- I can rewrite expression in rational exponent form and radical form
- I can evaluate radicals and rational exponents without a calculator
- I can evaluate radicals and rational exponents with a calculator
- I can simplify expressions with rational exponents

Rational exponent notation:	$a^{m/n}$	$a^{-m/n}$	$-a^{m/n}$	$(-a)^{m/n}$
Radical notation:	$\sqrt[n]{a^m}$	$\frac{1}{\sqrt[n]{a^m}}$	$-\sqrt[n]{a^m}$	$\sqrt[n]{(-a)^m}$

Rewrite the expression using radical notation.

Examples: $4^{5/2}$ $\qquad$ $-5^{1/2}$

$\sqrt{4^5}$ $\qquad$ $-\sqrt{5}$

Rewrite the expression using rational exponent notation.

Examples: $\sqrt[4]{-5}$ $\qquad$ $\sqrt[2]{6^3}$

$(-5)^{1/4}$ $\qquad$ $6^{3/2}$

Evaluate <u>with</u> a calculator.

$6^{1/3}$	$\sqrt[4]{10}$	$\sqrt[5]{-4^3}$	$-12^{3/4}$
1.817	1.778	−2.297	−6.447

Quick Check:

Rewrite the expression using radical notation and evaluate with a calculator.

1. $(-8)^{2/3}$

$\sqrt[3]{(-8)^2}$

2. $3^{-5/4}$

$\frac{1}{\sqrt[4]{3^5}}$

Rewrite the expression using rational exponent notation and evaluate with a calculator.

3. $\sqrt[5]{(-2)^4}$

$(-2)^{4/5}$

4. $-\sqrt{7^3}$

$-7^{3/2}$

Base	***2***	***3***	***4***	***5***	***6***	***7***
Perfect Squares	4	9	16	25	36	49
Perfect Cubes	8	27	64	125	216	343
Perfect 4^{th} powers	16	81	256	625	1296	
Perfect 5^{th} powers	32	243	1024	3125		

To evaluate:
- Change to radical form.
- Take the root (if even).
- Take the power.

Evaluate without using a calculator.

Examples: $8^{-4/3}$

$\frac{1}{\sqrt[3]{8^4}} = \frac{1}{2^4} = \boxed{\frac{1}{16}}$

$(-125)^{-2/3}$

$\frac{1}{\sqrt[3]{(-125)^2}} = \frac{1}{(-5)^2} = \boxed{\frac{1}{25}}$

$(81)^{-3/2}$

$\frac{1}{\sqrt{81}^3} = \frac{1}{9^3} = \boxed{\frac{1}{729}}$

$\sqrt[3]{(-64)^4}$

$(-4)^4$

$\boxed{256}$

Use the properties of rational exponents to simplify the expression.

Examples: $9^{1/2} \cdot 9^{3/4}$

$9^{1/2+3/4}$

$9^{2/4+3/4} = 9^{5/4}$

$(7^{2/3} \cdot 5^{1/6})^3$

$7^2 \cdot 5^{\frac{1}{2}}$

$\frac{42x^4y^7}{6x^{3/2}y^{-3}z^7}$

$\frac{42x^4y^7y^3}{6x^{3/2}z^7}$

$\frac{7x^4y^{10}}{x^{3/2}z^7} = \frac{7x^{5/2}y^{10}}{z^7}$

$(36m^4n^{10})^{1/2}$

$6m^2n^5$

$\sqrt[3]{\frac{a^6}{b^9}}$

$\left(\frac{a^6}{b^9}\right)^{1/3} = \frac{a^2}{b^3}$

$x^{1/3}(\sqrt[2]{x} \cdot y^3)^4$

$x^{1/3}(x^{1/2} \cdot y^3)^4$

$x^{1/3}\,x^2\,y^{12}$

$x^{7/3}y^{12}$

Tips for simplifying:

--Look for something that is the same
- If bases are the same, combine the powers. Ex. $3^{1/2} \cdot 3^{2/3} = 3^{1/2+2/3}$
- If powers are the same, combine the bases. Ex. $3^{1/2} \cdot 5^{1/2} = 15^{1/2}$

--Change radicals to rational exponents

--If problem seems too complex, think of a simpler example to know what to do with the exponents.

Ex. $x^{3/4} \cdot x^{2/3}$ Think $x^a \cdot x^b = x^{a+b}$
Then $\frac{3}{4} + \frac{2}{3} = \frac{9}{12} + \frac{8}{12} = \frac{17}{12}$
so $x^{3/4} \cdot x^{2/3} = x^{17/12}$

Do & Discuss:

5. $(6^6 \cdot 5^6)^{-1/6}$

$6^{-1} \cdot 5^{-1}$

$\frac{1}{6 \cdot 5} = \frac{1}{30}$

6. $4^{1/2} \cdot 4^{3/2}$

$4^{4/2} = 4^2 = 16$

7. $\frac{3x^{1/3}y}{(16y)^{-1/4}}$

$\frac{3x^{1/3}y}{16^{-1/4}y^{-1/4}} = 3x^{1/3}y\,16^{1/4}y^{1/4}$

$= 3x^{1/2}y^{5/4}\,2$

$= 6x^{1/2}y^{5/4}$