3.5: Perform Operations with Complex Numbers

I can simplify complex numbers
I can add, subtract, multiply and divide complex numbers
I can find complex solutions from quadratic equations

Complex Numbers ($a + bi$)

Real Numbers ($a + 0i$)	Imaginary Numbers ($a + bi, b \neq 0$)
	$2 + 3i$ $\quad 5 - 5i$
-1 $\quad \frac{5}{2}$	**Pure Imaginary Numbers** ($0 + bi, b \neq 0$)
π $\quad \sqrt{2}$	$-4i$ $\quad 6i$

Imaginary unit $i = \sqrt{-1} \quad i^2 = -1$

Complex number (in standard form): $a + bi$

Real Number: ~~$a + bi$~~, $b = 0$

Imaginary Number: $a + bi, b \neq 0$

Pure Imaginary Number: ~~a~~ $+ bi, a = 0, b \neq 0$

Write the expression as a complex number in standard form (a + bi):

Adding/Subtracting: combine the real parts and the imaginary parts.

Ex 1. $(8-i)+(5+4i)$

$8 - i + 5 + 4i$

$13 + 3i$

Ex 2. $(7-6i)-(3-6i)$

$7 - 6i - 3 + 6i$

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Multiplying: multiply, substitute i^2 with -1, combine like parts

Ex 3. $4i(-6+i)$

$-24i + 4i^2$

$-24i + 4(-1)$

$-4 - 24i$

Ex 4. $(9-2i)(-4+7i)$ foil

$-36 + 63i + 8i - 14i^2$

$-36 + 71i - 14(-1)$

$-36 + 71i + 14$

$-22 + 71i$

Quick Check

1. $10-(6+7i)+4i$

$10 - 6 - 7i + 4i$

$4 - 3i$

2. $-4-(1+i)-(5+9i)$

$-4 - 1 - i - 5 - 9i$

$-10 - 10i$

3. $(3+i)(5-i)$

$15 - 3i + 5i - i^2$

$15 + 2i + 1$

$16 + 2i$

Dividing: we do not want a complex number in the denominator so multiply by the conjugate

Complex Conjugates: $a+bi$ and $a-bi$

****when you multiply complex conjugates the product is always a** real **number**

Ex 5. $\frac{7+5i}{1-4i}$ $(1+4i)$ $(1+4i)$ $\frac{7+28i+5i+20i^2}{1+4i-4i-16i^2} = \frac{7+33i+20(-1)}{1-16(-1)} = \frac{-13+33i}{17}$

Solving Quadratics with Complex Answers

$\sqrt{-9} =$ $\sqrt{9}\sqrt{-1}$ $3i$

$\sqrt{-18} =$ $\sqrt{18}\sqrt{-1}$ $\sqrt{9}\sqrt{2}\,i$ $3\sqrt{2}\,i$

Find the solutions of the equations.

Ex 6. $2x^2 + 31 = 9$

$-31 \quad -31$

$\frac{2x^2}{2} = \frac{-22}{2}$

$\sqrt{x^2} = \sqrt{-11}$

$x = \pm i\sqrt{11}$

Ex 7. $x^2 + 6x = -15$

$x^2+6x+15=0$

$x = \frac{-6 \pm \sqrt{6^2-4(1)(15)}}{2(1)}$

$x = \frac{-6 \pm \sqrt{-24}}{2} = \frac{-6 \pm 2i\sqrt{6}}{2} = -3 \pm i\sqrt{6}$

Simplify: $\frac{4+2i}{3-2i}$ $3+2i$ $3+2i$

$\frac{12+8i+6i+4i^2}{9-4i^2}$

$\frac{8+14i}{13}$

Solve: $6x^2 + 4x + 11 = 0$

$x = \frac{-4 \pm \sqrt{4^2-4(6)(11)}}{2(6)}$

$x = \frac{-4 \pm \sqrt{-248}}{12} = \frac{-4 \pm 2\sqrt{62}\,i}{12}$

$= \frac{-2 \pm \sqrt{62}\,i}{6}$

Additional Resources: 4.6 from the Textbook