Key

3.4 Solving Quadratic Equations using the Quadratic Formula

- I can use the quadratic formula to solve a quadratic equation
- I can evaluate the discriminant and use it to determine the number and type of solutions

VOCABULARY

Standard form of a quadratic equation: $ax^2+bx+c=0$

Quadratic Formula: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Discriminant: b^2-4ac (The expression under the square root in the quadratic formula.)

Use the quadratic formula to solve. Simplify your answer.

*Make sure the quadratic equation is set equal to zero. Identify a, b and c before you begin.

Ex1: $x^2+3x=-2$ $\quad x^2+3x+2=0$

a= 1 b= 3 c= 2

$$x=\frac{-3\pm\sqrt{9-4(1)(2)}}{2(1)}=\frac{-3\pm\sqrt{9-8}}{2}$$

$$x=\frac{-3\pm1}{2}$$

$$\frac{-3+1}{2}=\frac{-2}{2}=-1$$

$$\frac{-3-1}{2}=\frac{-4}{2}=-2$$

$x=-1$ or -2

Ex2: $4x^2=8x-1$

$4x^2-8x+1=0$

a= 4 b= −8 c= 1

$$x=\frac{8\pm\sqrt{64-4(4)(1)}}{2(4)}=\frac{8\pm\sqrt{64-16}}{8}=\frac{8\pm\sqrt{48}}{8}$$

$$=\frac{8\pm\sqrt{16}\cdot\sqrt{3}}{8}=\frac{8\pm4\sqrt{3}}{8}=\frac{2\pm\sqrt{3}}{4}$$

Do and Discuss

Solve the equation and simplify your answer(s).

1. $8x^2-8x=-2$ $8x^2-8x+2=0$

a= 8 b= -8 c= 2

$x=\frac{8\pm\sqrt{64-4(8)(2)}}{2(8)}$

$=\frac{8\pm\sqrt{0}}{2}=4$

2. $3-8x-5x^2=2x$ $5x^2+10x-3=0$

a= 5 b= 10 c= -3

$x=\frac{-10\pm\sqrt{100-4(5)(-3)}}{2(5)}$

$=\frac{-10\pm\sqrt{160}}{10}=\frac{-10\pm4\sqrt{10}}{10}$

$x=\frac{-5\pm2\sqrt{10}}{5}$

Using the Discriminant – Looking under the square root can tell you a lot!

If b^2-4ac is:

positive → 2 **real solutions. Why?** Every positive # has 2 square roots

zero → 1 **real solutions. Why?** zero only has 1 squa ro

negative → 0 **real solutions. Why?** Can't take the squa root of a negati #(in real # syste

(We will discuss imaginary solutions soon!)

Determine the number of x-intercepts of the parabola. Hint: Remember that the x-intercepts correspond to the number of real solutions.

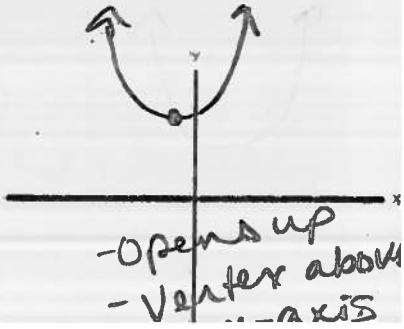
Ex 3: $4x^2+3x+12=3-3x$

$4x^2+6x+9=0$

a= 4 b= 6 c= 9, so $b^2-4ac=$ $36-4(4)(9)$

$36-144=-108$

Rough Sketch:



- opens up
- Vertex abov
- -axis

No x-intercepts
(No real solutions)

Quick Check:

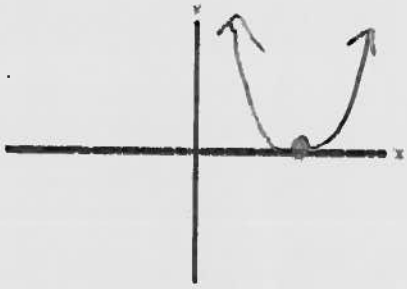
Determine the number of real solutions. Draw a rough sketch of the parabola.

1. $x^2-8x=-16$

$x^2-8x+16=0$

$b^2-4ac = 64-4(1)(16) = 64-64 = 0$

One real solution



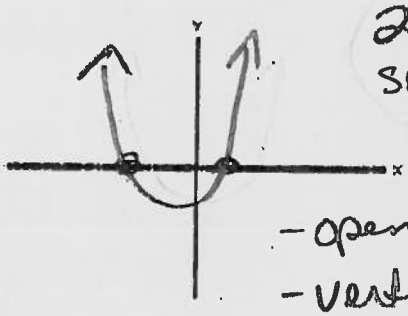
- opens up
- Vertex on x-axis

2. $2x^2+4x-4=0$

$b^2-4ac = 16-4(2)(-4)$

$16+32 = 48$

2 real solutions



- opens up
- vertex below x-axis

Ex 4: A juggler tosses a ball into the air. The ball leaves his hand 4 feet above the ground and has an initial velocity of 40 feet/second. The juggler catches the ball when it is 3 feet above the ground. How long was the ball in the air?

$h=-16t^2+v_0t+h_0$ Where h = height at time t. v_0 = initial velocity

h_0 = initial height

$3=-16t^2+40t+4$

$-16t^2+40t+1=0$

$$x=\frac{-40\pm\sqrt{1600-4(-16)(1)}}{2(-16)}=\frac{-40\pm\sqrt{1664}}{-32}=\frac{-40\pm 40.79}{-32}$$

$\frac{-40+40.79}{-32}$ $\frac{-40-40.79}{-32}$

$-.025$ negative

2.52 sec

Additional Resources:

- Textbook: Chapter 4.8 (pg. 292)
- The Quadratic Formula Song: https://youtu.be/O8ezDEk3qCg
- Quadratic Formula - Kahn Academy https://youtu.be/i7idZfS8t8w