

3.3 Solving Quadratic Equations by Factoring

- I can factor a quadratic equation
- I can solve a quadratic equation that has been factored

VOCABULARY

Monomial: A number, variable, or product of a number and variable(s)

Greatest Common Factor (GCF): The largest monomial that ALL terms can be divided by evenly.

Zero Product Property: If a product is zero, one of the factors must be zero.
If $a \cdot b = 0$, then $a = 0$ or $b = 0$.

Factor out the GCF

Ex1: $4x^3 - 18x^2 + 10x$

$$2x(2x^2 - 9x + 5)$$

Ex2: $3x^2y + 5xy^2$

$$xy(3x + 5y)$$

Solve using the Zero Product Property

Ex3: $0 = (x-3)(x+1)$

$$\begin{array}{ccc} x-3=0 & \text{or} & x+1=0 \\ +3 & +3 & -1 & -1 \end{array}$$

$$\boxed{x=3, \quad x=-1}$$

Ex4: $4x(3x+5) = 0$

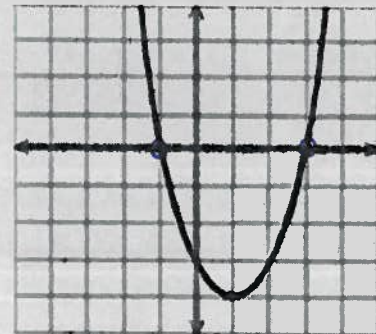
$$\begin{array}{ccc} \frac{4x}{4} = \frac{0}{4} & \text{or} & 3x+5=0 \\ & & -5 & -5 \end{array}$$

$$\boxed{x=0}$$

$$\frac{3x}{3} = \frac{-5}{3}$$

$$\boxed{x = -\frac{5}{3}}$$

Graphing Connection



Solving a quadratic equation set equal to zero is the same as finding the x-intercepts of the graph.

SOLVING QUADRATICS BY FACTORING:

Step 1: Factor out the GCF first!!! Make sure a is POSITIVE!

Step 2: Factor using your preferred method...

- a. Trial & Error
- b. The Box
- c. The X Puzzle (Railroad)

Step 3: Solve using Zero Product Property

METHODS OF FACTORING:

A. Trial & Error $2x^2 - 13x - 24$

Handwritten trial and error work for $2x^2 - 13x - 24$:

- $1 \cdot 2$
- $2 \cdot 12$
- $3 \cdot 8$
- ~~$4 \cdot 6$~~

Factor pairs for -24 :

- $1 \cdot 24$
- $2 \cdot 12$
- $3 \cdot 8$
- ~~$4 \cdot 6$~~

Final factored form: $(x - 8)(2x + 3)$

B. The Box $2x^2 - 13x - 24$

Box method diagram for $2x^2 - 13x - 24$:

$2x$	$2x^2$	
x		-24

Labels: $-13x$ (pointing to the middle of the box), and a list of factor pairs for -24 on the right: $1 \cdot 24, 2 \cdot 12, 3 \cdot 8, 4 \cdot 6$.

C. The X Puzzle $2x^2 - 13x - 24$
 $a=2 \quad b=-13 \quad c=-24$

" a times c add to b divide by a ... reduce"

1. Fill in the X
 - a. Top: What is a times c ?
 - b. Bottom: What is b ?
2. List factor pairs for $a \cdot c$
3. Which pair adds/subtracts to b ?
4. Fill in the left and right of the X with that pair.
5. Figure out your signs! Make sure it works!
6. Divide your answers by a
7. Reduce if you can
8. Write your factors

denominator \rightarrow coefficient of x
numerator \rightarrow + or - from x

Handwritten X puzzle diagram for $2x^2 - 13x - 24$:

- Top: $a \cdot c = -48$
- Bottom: -13
- Left: $+\frac{3}{2}$
- Right: $-\frac{16}{2} = -8$

Factor pairs for -48 are listed on the right: $+48, -24, 3 \cdot 16$ (circled).

Final factored form: $(2x + 3)(x - 8)$

- ① set = 0
- ② GCF
- ③ Factor
- ④ Solve

3.3 Solve Quadratics by Factoring

Solve by Factoring

Ex5: $0 = 2x^2 - 16x + 24$

$$0 = 2(x^2 - 8x + 12)$$

$$0 = 2(x-2)(x-6)$$

$$\begin{array}{l} x-2=0 \\ +2 \quad +2 \end{array} \quad \begin{array}{l} x-6=0 \\ +6 \quad +6 \end{array}$$

$$\boxed{x = 2, x = 6}$$

$a=1 \quad b=-8 \quad c=12$

$$\begin{array}{r} 12 \\ -2 \quad -6 \\ \hline -8 \end{array}$$

Ex6: $x^2 + 3x - 28 = 0$

$$(x-4)(x+7) = 0$$

$$\begin{array}{l} x-4=0 \\ +4 \quad +4 \end{array} \quad \begin{array}{l} x+7=0 \\ -7 \quad -7 \end{array}$$

$$\boxed{x = 4, x = -7}$$

$a=1 \quad b=3 \quad c=-28$

$$\begin{array}{r} -28 \\ -4 \quad 7 \\ \hline 3 \end{array}$$

Quick Check:

A: $3 = 2x^2 - x$

$$0 = 2x^2 - x - 3$$

$$0 = (x+1)(2x-3)$$

$$\begin{array}{l} x+1=0 \\ \frac{2x-3=0}{\frac{2x}{2} = \frac{3}{2}} \end{array}$$

$$\boxed{x = -1}$$

$$\frac{2x}{2} = \frac{3}{2}$$

$$\boxed{x = \frac{3}{2}}$$

-6

$$\begin{array}{r} -6 \\ +2 \quad -3 \\ \hline -1 \end{array}$$

B: $0 = 8x^2 + 4x$

$$0 = 4x(2x+1)$$

$$\begin{array}{l} 4x=0 \\ \frac{4x}{4} = \frac{0}{4} \end{array} \quad \begin{array}{l} 2x+1=0 \\ \frac{2x}{2} = \frac{-1}{2} \end{array}$$

$$\boxed{x = 0}$$

$$\frac{2x}{2} = \frac{-1}{2}$$

$$\boxed{x = -\frac{1}{2}}$$

Do & Discuss

A town has a nature preserve with a rectangular field that measures 600 meters by 400 meters. The town wants to double the area of the field by adding land as shown. Find the new dimensions of the field.

$$400 \cdot 600 = 240,000 \times 2 = 480,000$$

$$(x+400)(x+600) = 480,000$$

$$x^2 + 600x + 400x + 240,000 = 480,000$$

$$x^2 + 1000x - 240,000 = 0$$

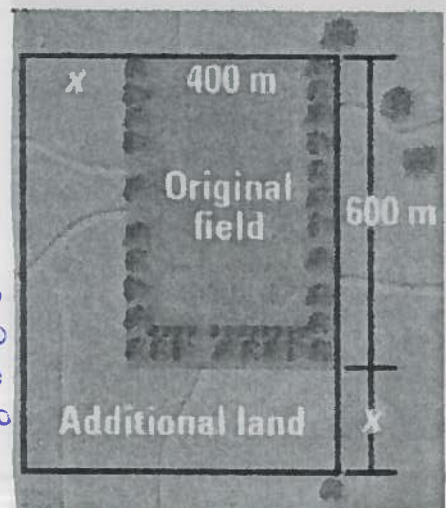
$$(x-200)(x+1200) = 0$$

$$\boxed{x = 200}, -1200$$

$$\boxed{800 \text{ m} \times 600 \text{ m}}$$

$-240,000$

$$\begin{array}{r} -240,000 \\ -200 \quad 1200 \\ \hline 1000 \end{array}$$



Additional Resources:

- Textbook: Chapter 4.3 (pg. 252) and 4.4 (pg.259)
- Other Method of Factoring Video: <https://www.khanacademy.org/math/algebra2/polynomial-functions/factoring-polynomials-quadratic-forms-alg2/v/factoring-trinomials-by-grouping-6>
- Online Practice: <https://www.khanacademy.org/math/algebra-basics/quadratics-polynomials-topic/solving-quadratics-factoring/e/solving-quadratics-by-factoring-2>