3.2: Solving Quadratic Equations Using Square Roots

Key

- I can simplify radical expressions
- I can solve a quadratic equation using square roots

Vocabulary

radical → $\sqrt{x}$ ← radicand	*a* is the **square root** of *b* if $a^2 = b$ Ex: 4 is the square root of 16 since $4^2 = 16$

Every number has 2 square roots. One is positive and one is negative.

The symbol for the positive square root is $\sqrt{\ }$ and the symbol for the negative is $-\sqrt{\ }$

Properties of Square Roots

Product Property: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ Example: $\sqrt{18} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$

Quotient Property: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ Example: $\sqrt{\frac{2}{25}} = \frac{\sqrt{2}}{\sqrt{25}} = \frac{\sqrt{2}}{5}$

Do and Discuss

Simplify the expression:

a. $\sqrt{80} = \sqrt{16} \cdot \sqrt{5}$ $4\sqrt{5}$

b. $\sqrt{\frac{4}{81}}$ $\frac{\sqrt{4}}{\sqrt{81}} = \frac{2}{9}$

c. $\sqrt{6} \cdot \sqrt{2} = \sqrt{12}$ $= \sqrt{4} \cdot \sqrt{3}$ $2\sqrt{3}$

d. $\sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{\sqrt{16}}$ $\sqrt{7}/4$

Rationalizing the Denominator:

If there is an irrational number in the denominator, we want to rationalize it.

If the denominator is in the form $\sqrt{b}$ multiple both numerator and denominator by $\sqrt{b}$.

If the denominator is in the form $a+\sqrt{b}$ multiple both numerator and denominator by $a-\sqrt{b}$.

If the denominator is in the form $a-\sqrt{b}$ multiple both numerator and denominator by $a+\sqrt{b}$.

Conjugates: $a+\sqrt{b}$ and $a-\sqrt{b}$ are called conjugates. (Just opposites)

When you multiply two conjugates, the result is a rational number.

Ex 1: Simplify the expression (remove any square roots from the denominator).

a. $\sqrt{\frac{5}{2}} = \frac{\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{10}}{2}$

b. $\frac{3}{(7+\sqrt{2})} \frac{(7-\sqrt{2})}{(7-\sqrt{2})} = \frac{21-3\sqrt{2}}{49-7\sqrt{2}+7\sqrt{2}-\sqrt{2}\cdot\sqrt{2}}$

$= \frac{21-3\sqrt{2}}{49-2} = \frac{21-3\sqrt{2}}{47}$

Solving Quadratic Equations

Solve and simplify your answer

Steps
1. Get the squared factor alone
2. Take the square root (remember there are 2 !)
3. Solve for x.

Ex 2: $3x^2+5=41$

$-5 \quad -5$

$\frac{3x^2}{3} = \frac{36}{3}$ $\quad x^2=12$ $\quad x = \pm\sqrt{12}$

$= \pm\sqrt{4}\cdot\sqrt{3}$

$x = \pm 2\sqrt{3}$

Ex 3: $\frac{3(x-2)^2}{3} = \frac{40}{3}$

$(x-2)^2 = \frac{40}{3}$ $\quad x-2 = \pm\sqrt{\frac{40}{3}}$ $\quad x-2 = \pm\frac{\sqrt{40}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\pm\sqrt{120}}{3}$

$x-2 = \frac{\pm\sqrt{4}\cdot\sqrt{30}}{3}$ $\quad x = \frac{2\pm 2\sqrt{30}}{3}$

Do and Discuss:

Simplify

1. $\sqrt{48}$

$\sqrt{16}\cdot\sqrt{3}$

$4\sqrt{3}$

2. $\frac{5\sqrt{8}}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}$

$\frac{5\sqrt{40}}{5} = \sqrt{4}\cdot\sqrt{10}$

$2\sqrt{10}$

3. $\frac{4}{3-\sqrt{5}}\frac{(3+\sqrt{5})}{(3+\sqrt{5})}$

$= \frac{12+4\sqrt{5}}{9-5} = \frac{12+4\sqrt{5}}{4} = 3+\sqrt{5}$

Solve and simplify your answer

1. $5x^2+1=61$

$-1 \quad -1$

$\frac{5x^2}{5} = \frac{60}{5}$

$x^2=12$

$x = \pm\sqrt{12}$

$x = \pm 2\sqrt{3}$

2. $-5(x+6)^2+3=-1$

$-3 \quad -3$

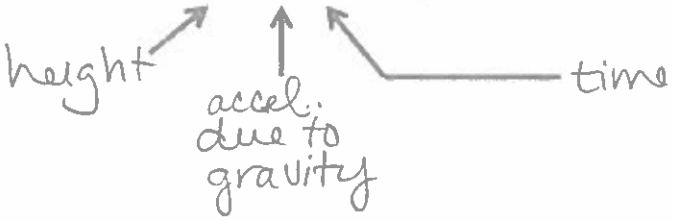
$\frac{-5(x+6)^2}{-5} = \frac{-4}{-5}$

$(x+6)^2 = \frac{4}{5}$ $\quad x+6 = \pm\frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}$

$x = -6 \pm \frac{2\sqrt{5}}{5}$

Modeling a Dropped Object:

When an object is dropped, the height (h) in feet of the object after t seconds can be modeled by the function $h = -16t^2 + h_0$ ← initial height



You drop a water balloon off the roof of your apartment building that is 240 feet tall. About how long will it take for the balloon to hit the head of your friend that is 6 feet tall?

$6 = -16t^2 + 240$

$-240 \quad -240$

$-234 = -16t^2$

$\frac{-234}{-16} = \frac{-16t^2}{-16}$

$t^2 = 14.625$

$t = \pm\sqrt{14.625}$

$t = 3.82$ sec (can't be negative)

If you missed your friend, how long does it take for the balloon to hit the ground?

$0 = -16t^2 + 240$

$\frac{-240}{-16} = \frac{-16t^2}{-16}$

$15 = t^2$

$t = \pm\sqrt{15}$

$t = 3.87$ sec

Additional Resources: Textbook: Section 4.5 starts on pg. 266