(Key)

2.3: *Operations with Functions*

- I can add, subtract and multiply functions
- I can divide functions by factoring
- I can perform composition of functions

Operation	Definition	Example: *f(x)* = 5x, *g(x)* = x + 2	Domain
Addition	*h(x) = f(x) + g(x)*	$h(x)=5x+x+2=6x+2$	$\mathbb{R}$
Subtraction	*h(x) = f(x) - g(x)*	$h(x)=5x-(x+2)=4x-2$	$\mathbb{R}$
Multiplication	*h(x) = f(x) • g(x)*	$h(x)=5x(x+2)=5x^2+10x$	$\mathbb{R}$
Division	$h(x)=\frac{f(x)}{g(x)}$	$h(x)=\frac{5x}{x+2}$	$x\neq-2$

Adding and Subtracting Functions

Remember: Like Terms have the same variables with the same exponents

Do and Discuss

For the functions $f(x)=3x^2-5x+10$, $g(x)=x^2-6$, $h(x)=8-4x$ evaluate and simplify:

1. $f(x)+g(x)$

$3x^2-5x+10+x^2-6$

$4x^2-5x+4$

2. $h(x)+g(x)=8-4x+x^2-6$

$=x^2-4x+2$

3. $g(x)-f(x)$

$x^2-6+(-3x^2+5x-10)$

$=-2x^2+5x-16$

4. $h(x)-f(x)=8-4x+(-3x^2+5x-10)$

$=-3x^2+x-2$

Multiplying Functions

To multiply functions, use the distributive property.

$p(x)=-5x^3$, $r(x)=2x^2+4x-3$, $q(x)=9-7x$

Ex 1: Evaluate and simplify $p(x)\bullet r(x)$

$-5x^3(2x^2+4x-3)=-10x^5-20x^4+15x^3$

Ex 2: Evaluate and simplify $q(x)\bullet r(x)$

$(9-7x)(2x^2+4x-3)=18x^2+36x-27-14x^3-28x^2+21x$

$=-14x^3-10x^2+57x-27$

Quick Check:

$$f(x) = 2x + 1 \qquad g(x) = 4x^2 + x \qquad h(x) = x^3 - 5x$$

A. $f(x)+h(x)$

$2x+1+x^3-5x$

x^3-3x+1

B. $h(x)-f(x)$

$x^3-5x-(2x+1)$

x^3-7x-1

C. $g(x)\cdot f(x)\cdot h(x)$

$(4x^2+x)(2x+1)(x^3-5x)$

$8x^3+4x^2+2x^2+x$

$(8x^3+6x^2+x)(x^3-5x)$

$8x^6+6x^5+x^4-40x^4-30x^3-5x^2$

$8x^6+6x^5-39x^4-30x^3-5x^2$

Dividing Functions

Dividing is the inverse of multiplication. We use factoring to divide - which is the inverse of the distributive property. (We will do more factoring in Unit 3).

Ex 3: Find $\frac{f(x)}{h(x)}$ if $f(x)=-8x^3+6x^2+2x$ and $h(x)=2x$.

$$\frac{-8x^3+6x^2+2x}{2x} = \frac{-8x^3}{2x} + \frac{6x^2}{2x} + \frac{2x}{2x} = -4x^2+3x+1$$

Ex 4: Find $\frac{r(x)}{p(x)}$ if $r(x)=12x^6-48x^5+60x^2$ and $p(x)=-6x^2$.

$$\frac{12x^6}{-6x^2} \frac{-48x^5}{-6x^2} \frac{+60x^2}{-6x^2} = -2x^4+8x^3-10$$

Ex 5: Find $\frac{t(x)}{q(x)}$ if $t(x)=-x+5$ and $q(x)=2x-4$. $= \frac{-x+5}{2x-4}$

There are no common factors to divide out. Be careful that the denominator that remains does not have a value of zero. We must restrict that value from our domain.

The answer to the division problem is $\frac{-x+5}{2x-4}$ and the domain is $x \neq 2$.

Quick Check: If $g(x)=20x^4-16x^2+4x$, $h(x)=4x$, and $f(x)=2x-5$

A. Find $\frac{g(x)}{h(x)} = \frac{20x^4-16x^2+4x}{4x}$ Answer $5x^3-4x+1$ Domain $\mathbb{R}$

B. Find $\frac{h(x)}{f(x)} = \frac{4x}{2x-5}$ Answer $\frac{4x}{2x-5}$ Domain $x \neq 5/2$

Composition of Functions

A function in which the output of one function becomes the input for another. (Putting one function inside of another function).

The composition of the function *f* with the function *g* is $h(x) = f(g(x))$. You say "f of g of x"

The **domain** of h(x) includes all the x-values in the domain of g as well as values of g(x) in the domain of f.

Ex 6: $f(x) = 4x$ and $g(x) = 3x+5$

a. Find f(g(5))

$g(5) = 3(5)+5 = 20$

$f(20) = 4 \cdot 20 = 80$

b. Find g(f(2))

$f(2) = 4(2) = 8$

$g(8) = 3(8)+5 = 24+5 = 29$

c. Find g(f(x))

$g(4x) = 3(4x)+5 = 12x+5$

d. Find f(g(x))

$f(3x+5) = 4(3x+5) = 12x+20$

e. What is the domain of g(f(x))? $\mathbb{R}$

f. Domain of f(g(x))? $\mathbb{R}$

Domain: $\mathbb{R}$ $\quad x \neq 0 \quad x \geq 0$

Ex 7: If $f(x) = 2x$, $h(x) = \frac{1}{x}$, $g(x) = \sqrt{x}$ find the following composite function and state its domain.

a. f(h(x))= $f(\frac{1}{x}) = 2(1/x) = \frac{2}{x}$ $\quad$ Domain $x \neq 0$

b. h(g(x))= $h(\sqrt{x}) = \frac{1}{\sqrt{x}}$ $\quad$ Domain $x > 0$

Do & Discuss:

Domain: $x \neq 0 \quad \mathbb{R} \quad \mathbb{R}$

Given $t(x) = \frac{1}{x}$, $r(x) = 3x-1$, and $q(x) = x^2$, find:

5. r(t(1/2))

$t(1/2) = \frac{1}{1/2} = 2$

$r(2) = 3(2)-1 = 5$

6. r(q(-1))

$q(-1) = (-1)^2 = 1$

$r(1) = 3(1)-1 = 2$

7. q(r(x)) $\quad$ Domain: $\mathbb{R}$

$q(3x-1) = (3x-1)^2 = 9x^2-6x+1$

8. t(r(x)) $\quad$ Domain: $x \neq 1/3$

$t(3x-1) = \frac{1}{3x-1}$

Additional Resources: Textbook: Section 5.3 starts on pg. 346 and Section 6.3 starts on pg.428