Chapter 4: Matrices

- I can add and subtract matrices of the same dimension
- I can multiply a matrix by a scalar
- I can multiply matrices

VOCABULARY

Matrix: A rectangular arrangement of numbers into rows and columns.

Elements: The numbers in a matrix

Matrix Dimensions: (m x n)

m = number of rows (horizontal)

n = number of columns (vertical)

columns ↕

$$A = \underbrace{\begin{bmatrix} 3 & -2 & 0 \\ 5 & 8 & -4 \end{bmatrix}}_{\text{dimension: } 2\times 3} \Big\} \text{rows} \leftrightarrow$$

elements

Scalar: A real number that we usually multiply by. (Very much like "Scale Factor" in Geometry.)

Adding & Subtracting Matrices:

- Add or subtract corresponding values (numbers in the same spot).
- Subtraction: Change to adding the opposite.
  Or just be extremely careful!!!

Ex 1: $\begin{bmatrix} 3 & -2 & 0 \\ 5 & 8 & -4 \end{bmatrix} + \begin{bmatrix} 7 & 2 & -8 \\ 3 & -6 & -1 \end{bmatrix}$

$= \begin{bmatrix} 10 & 0 & -8 \\ 8 & 2 & -5 \end{bmatrix}$

Ex 2: $\begin{bmatrix} 7 & -12 \\ 3 & -5 \end{bmatrix} - \begin{bmatrix} -6 & 8 \\ 4 & -2 \end{bmatrix}$

$= \begin{bmatrix} 13 & -20 \\ -1 & -3 \end{bmatrix}$

Scalar Multiplication:

- Multiply every element by the number.
- Do this BEFORE adding/subtracting matrices.

Ex 3: $-3\begin{bmatrix} 2 & 0 \\ -1 & -5 \end{bmatrix} = \begin{bmatrix} -6 & 0 \\ 3 & 15 \end{bmatrix}$

Quick Check:

Simplify into a single matrix.

$$4\begin{bmatrix} -1 & 6 \\ 3 & -5 \\ -2 & 7 \end{bmatrix} - 5\begin{bmatrix} 0 & -3 \\ 8 & 4 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} -4 & 24 \\ 12 & -20 \\ -8 & 28 \end{bmatrix} + \begin{bmatrix} 0 & 15 \\ -40 & -20 \\ 15 & -25 \end{bmatrix} = \begin{bmatrix} -4 & 39 \\ -28 & -40 \\ 7 & 3 \end{bmatrix}$$

Matrix Multiplication: [Matrix A] X [Matrix B] = [Matrix AB]

CAN YOU MULTIPLY?

- **Defined when the #COLUMNS of A = #ROWS of B. (If not, you can't do it→ undefined!)**
- **The resulting matrix dimension is the ROWS of A x COLUMNS of B**

Trick: Write the 2 dimensions side-by-side in order. • If the **middle numbers match**, it's defined. • The **dimension** of the resulting matrix is the **outer numbers.**	Look at A and B below: Can we find AB? A B 2×2 2×3 Yes → 2×3 Can we find BA? B A 2×3 2×2 NO!

HOW TO MULTIPLY: RowsA x ColumnsB

1. Multiply each number in ROW1 of A by the corresponding number in COLUMN1 of B
   - o (1^{st} x 1^{st}, 2^{nd} x 2^{nd}, 3^{rd} x 3^{rd}, etc.)
   - o ADD up these values. This gives you the element for Row1 Column1
2. Multiply each number in Row1 of A by the corresponding number in Column2 of B
   - o Repeat until all columns of B have been used
3. Move to Row2 of A and repeat steps 1 & 2
4. Continue until all Rows of A have multiplied by all Columns of B

$A=\begin{bmatrix}3 & -1\\ 2 & -3\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 0 & 5\\ 4 & -2 & 6\end{bmatrix}$ AB = $\begin{bmatrix}(3)(-1)+(-1)(4) & 3(0)+(-1)(-2) & (3)(5)+(-1)(6)\\ 2(-1)+(-3)(4) & 2(0)+(-3)(-2) & (2)(5)+(-3)(6)\end{bmatrix}$

$$= \begin{bmatrix}-3-4 & 0+2 & 15-6\\ -2-12 & 0+6 & 10-18\end{bmatrix} = \begin{bmatrix}-7 & 2 & 9\\ -14 & 6 & -8\end{bmatrix}$$

Do & Discuss:

Multiply if possible.

$C=\begin{bmatrix}-2 & -6\\ 0 & -1\end{bmatrix}$, $D=\begin{bmatrix}1 & 0\\ -3 & 2\\ 6 & -4\end{bmatrix}$, $E=\begin{bmatrix}-0.5 & 3\\ 0 & -1\end{bmatrix}$

2×2 3×2 2×2

1. CD undefined

2. DC $= \begin{bmatrix}-2+0 & -6+0\\ 6+2 & 18-2\\ -12+0 & -36+4\end{bmatrix} = \begin{bmatrix}-2 & -6\\ 8 & 16\\ -12 & -32\end{bmatrix}$

3. CE $= \begin{bmatrix}1+0 & -6+6\\ 0+0 & 0+1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ → 2×2 Identity matrix (C + E are inverses)

Additional Resources:

- Textbook 3.5 & 3.6 (begins on page 187)
- Videos and practice on Matrices: Go to the link, select a topic from Basic Matrix Operations or Matrix Multiplication. https://www.khanacademy.org/math/precalculus/precalc-matrices