1.3: Writing and Solving Systems of Linear Equations

- I can change words into symbols to write a system of equations
- I can solve a system of equations by graphing
- I can solve a system of equations by substitution
- I can solve a system of equations by elimination
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VOCABULARY

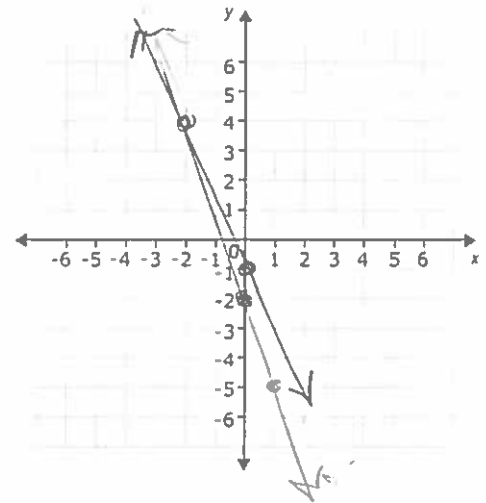| System of Linear Equations	Solution of the System
Two or more linear equations	The ordered pair (x, y) that make both equations true.

Warm Up: Solve the linear system by **graphing**

$y = -3x - 2$
$5x + 2y = -2$

$\dfrac{2y}{2} = \dfrac{-5x}{2} - \dfrac{2}{2}$

$y = -\dfrac{5}{2}x - 1$

Solution: $(-2, 4)$

Solving Systems Using Substitution

1. Solve one of the equations for one of its $\underline{\text{variables}}$ (get either x or y alone in one equation)
2. Substitute the expression into the other equation for the variable that you solved for.
3. Solve the equation and use that value to find the value of the other variable.

Ex. 1: Solve the system using **substitution**:

$2x + 5y = -5$
$x + 3y = 3$ $\qquad x = \boxed{3 - 3y}$

$2(3 - 3y) + 5y = -5$
$6 - 6y + 5y = -5$
$6 - y = -5$
$\boxed{y = 11}$

$x = 3 - 3 \cdot 11$
$x = 3 - 33$
$\boxed{x = -30}$

$\boxed{(-30, 11)}$

Quick Check: Solve by substitution.

$3x - y = -4$
$x + 3y = -28$ $\qquad x = \boxed{-28 - 3y}$

$3(-28 - 3y) - y = -4$
$-84 - 9y - y = -4$
$-10y = 80$
$\boxed{y = -8}$

$x = -28 - 3(-8)$
$x = -28 + 24$
$\boxed{x = -4}$ $\quad \boxed{(-4, -8)}$

Solving Systems by Elimination

1. Multiply one or both equations by a _Constant_ so that a set of coefficients are _opposites_
2. _Add_ the equations together to _eliminate_ one set of variables.
3. Solve for the remaining variable and use it to find the other _variable_.

Ex. 2: Solve the system using **elimination**

a.
$$2x - 3y = -1$$
$$2x + 3y = -19$$

$2(-5) - 3y = -1$

$-10 - 3y = -1$

$4x = -20$ $-3y = 9$

$x = -5$ $y = -3$

$(-5, -3)$

b.
$$-2(2x - 3y = 8) \quad -4x + 6y = -16$$
$$4x - 5y = 10 \quad 4x - 5y = 10$$

$y = -6$

$2x - 3(-6) = 8$

$2x + 18 = 8$ $(-5, -6)$

$2x = -10$

$x = -5$

Quick Check: Solve by elimination.

$$x + 2y = -8$$
$$3x - 4y = -24$$

$(-8, 0)$

Do and Discuss:

1. Solve by substitution:
$$x - 2y = -6$$
$$4x + 6y = 4$$

$(-2, 2)$
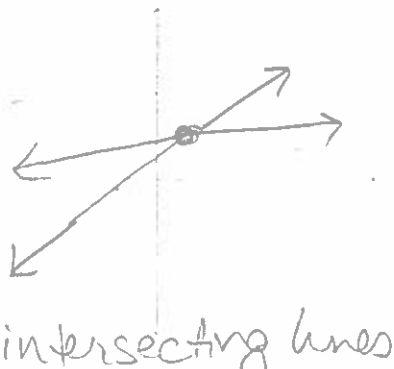
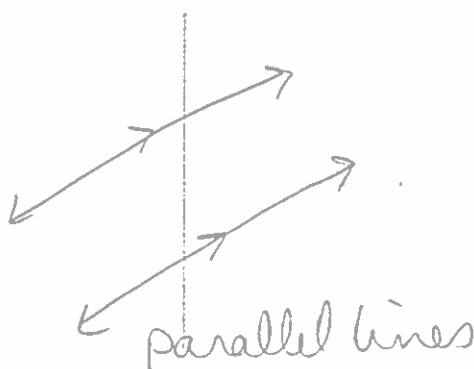2. Solve by elimination:
$$6x + 5y = 19$$
$$2x + 3y = 5$$

$(4, -1)$

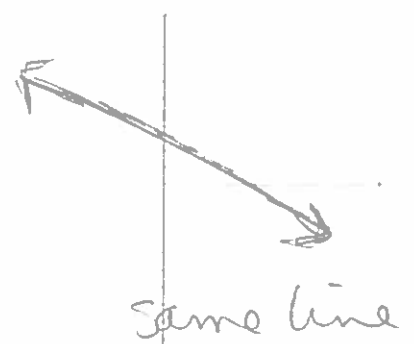3. Sketch 2 lines that form a system with the given number of solutions:

a. One solution

b. No solution

c. Infinitely many solutions

intersecting lines

parallel lines

same line

4. Write a system of two linear equations that has the given number of solutions
   (Hint: consider the graphs from #3 and write the equations in slope intercept form):

a. One solution

b. No solution

c. Infinitely many solutions

Ex

$$y = \frac{2}{3}x + 5$$
$$y = \frac{1}{2}x - 2$$

$$y = 5x - 2$$
$$y = 5x + 7$$

$$y = 3x - 1$$
$$2y = 6x - 2$$

Solving Special Types of Linear Systems:

Ex 3: Solve
$$12x + 4y = 6$$
$$6x + 2y = -5$$

When the variables _Cancel_ _out_

*And the remaining statement is _false_

⟹ _no solution_

(lines are parallel)

Ex 4: Solve
$$5x + y = -2$$
$$-10x - 2y = 4$$

*And the remaining statement is _true_

⟹ _infinitely many solut_

(same line)

Ex 5: Writing and solving a system of equations

You worked 14 hours last week and earned a total of $96 before taxes. Your job as a lifeguard pays $8 per hour and your job as a cashier pays $6 per hour. How many hours did you work at each job?

Let x= _# hrs lifeguarding_ and y = _# hrs cashiering_

Write one equation relating _hours_ and one equation relating _earnings_.

Do and Discuss:

Write the system of equations and solve.

5. You ride the bus to get from the center of town to your street. You have 2 payment options. Option A is to buy a $30 monthly pass and pay $1 per ride. Option B is to pay $2.50 per ride. After how many rides will the costs be the same?

Cost of Option A: $\underline{y = 30 + x}$

Cost of Option B: $\underline{y = 2.5x}$

$y =$ cost of ride
$x = \#$ rides

$30 + x = 2.5x$
$30 = 1.5x$
$\boxed{x = 20}$

If you ride 20 times the cost will be the same.

6. A science museum charges different admission rates for adults and students. Admission to the museum for 28 students and 5 adults costs $284. Admission for 40 students and 10 adults costs $440. What is the admission cost for one adult?

$a =$ cost of adult admiss.
$s =$ cost of student admiss.

$-2(28s + 5a = 284)$
$40s + 10a = 440$

$-56s - 10a = -568$
$\underline{40s + 10a = 440}$
$-16s = -128$
$\boxed{s = 8}$

$28(8) + 5a = 284$
$224 + 5a = 284$
$5a = 60$
$\boxed{a = 12}$

$\boxed{\text{Adult admiss. } \$12}$

Solve the system using **elimination**. (You will have to multiply both equations by a constant).

7.
$4x - 3y = 8$
$5x - 2y = -11$

$(-7, -12)$

8.
$7x - 6y = -1$
$5x - 4y = 1$

$(5, 6)$