

1.3 Enrichment: Solving Systems with 3 variables

Follow the example below and then solve the 3 problems within your group.

Strategy for solving: Use elimination twice to cancel one of the variables. Then you have 2 new equations with 2 variables.

| | | |
|--------------------------|----------------------|-------------------|
| Solve the system. | $4x + 2y + 3z = 1$ | Equation 1 |
| | $2x - 3y + 5z = -14$ | Equation 2 |
| | $6x - y + 4z = -1$ | Equation 3 |

Solution

STEP 1 Rewrite the system as a linear system in *two* variables.

| | |
|-----------------------|--------------------------------|
| $4x + 2y + 3z = 1$ | Add 2 times Equation 3 |
| $12x - 2y + 8z = -2$ | to Equation 1. |
| $16x + 11z = -1$ | New Equation 1 |
| $2x - 3y + 5z = -14$ | Add -3 times Equation 3 |
| $-18x + 3y - 12z = 3$ | to Equation 2. |
| $-16x - 7z = -11$ | New Equation 2 |

STEP 2 Solve the new linear system for both of its variables.

| | |
|-------------------|---|
| $16x + 11z = -1$ | Add new Equation 1 |
| $-16x - 7z = -11$ | and new Equation 2. |
| $4z = -12$ | |
| $z = -3$ | Solve for z. |
| $x = 2$ | Substitute into new Equation 1 or 2 to find x. |

STEP 3 Substitute $x = 2$ and $z = -3$ into an original equation and solve for y .

| | |
|-------------------------|---|
| $6x - y + 4z = -1$ | Write original Equation 3. |
| $6(2) - y + 4(-3) = -1$ | Substitute 2 for x and -3 for z. |
| $y = 1$ | Solve for y. |

- The solution is $x = 2$, $y = 1$, and $z = -3$, or the ordered triple $(2, 1, -3)$. Check this solution in each of the original equations.

To summarize, the 1st and 3rd equations were combined to eliminate the y , then the 1st and 2nd equations were combined to eliminate the y as well. Then elimination was used to solve for x and z . (You could do substitution too.) Those answers were used to solve for y .

1. $x + y - 5z = -5$

$y - 2z = 14$

$4y - 2z = 8$

2. $x - 2y + z = -1$

$x + 2y - z = 7$

$x + y + z = 2$

3. **Harvest Earning:** The feed mill pays a farmer \$6930.00 for the 1st delivery, \$5475.—for the 2nd delivery, and \$8879.50 for the 3rd delivery. The table shows the number of bushels included in each delivery. Use the table to write and solve a system of equation to find the price per bushel that the farmer received for each crop.

| Delivery | Corn | Wheat | Soybeans |
|--------------------------|-------------|--------------|-----------------|
| 1 st Delivery | 900 | 540 | 360 |
| 2 nd Delivery | 1125 | 150 | 225 |
| 3 rd Delivery | 860 | 645 | 645 |