Chapter 2: Absolute Value Equations and Inequalities

- I can solve an absolute value equation
- I can solve an absolute value inequality
- I can graph the solution to an absolute value inequality

VOCABULARY

Absolute Value	**Extraneous Solution**
The distance a # is from zero. (Always positive)	A "solution" that does not work in original equation (must be rejected)

Solving Absolute Value Equations

The absolute value of a number is always positive.

Ex 1: $|3| =$ 3 $\quad |-3| =$ 3

Ex 2: Solve $|x| = 5$ Think... What #'s are 5 units away from zero?

Solution(s): 5 and −5

Ex 3: Solve $|x| = -7$ Solution: No Solution because abs. value can't be negative.

Ex 4: $|2x-9|-5=10$ "what's inside abs. value could be pos. or neg."

$|2x-9| = 15$

$2x-9=15$ $\quad 2x-9=-15$

$2x=24$ $\quad 2x=-6$

$x=12$ $\quad x=-3$

Check:

$x=12$: $|2\cdot12-9|-5=10$

$|24-9|-5=10$ ✓

$x=-3$: $|2(-3)-9|-5=10$

$|-6-9|-5=10$ ✓

Solving Absolute Value Equations

1. Isolate abs. value symbols
2. Break into 2 equations (pos & negative)
3. Solve each equation
4. Check for extraneous solutions

Ex 5: $|4x+10| = 6x$

$4x+10=6x$ $\quad 4x+10=-6x$

$10=2x$ $\quad 10=-10x$

$x=5$ $\quad x=-1$ Extraneous solution

Check:

$|20+10| = 6\cdot5$

$30 = 30$ ✓

Check:

$|-4+10| = 6(-1)$

$6 \neq -6$

Quick Check: Solve and graph $3|4x-5| \le 12$ "and"

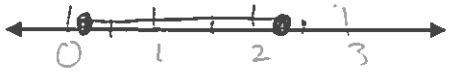
$|4x-5| \le 4$

$4x-5 \le 4$ and $4x-5 \ge -4$

$4x \le 9$ $\quad 4x \ge 1$

$x \le 9/4$ and $x \ge 1/4$

$\frac{1}{4} \le x \le 9/4$



Do and Discuss:

4. $2|x+1| \ge -32$ "or"

$|x+1| \ge -16$

$x+1 \ge -16$ or $x+1 \le 16$

$x \ge -17$ or $x \le 15$

All real #'s

5. $|2x-1|-5 < -30$ "and"

$|2x-1| < -25$

$2x-1 < -25$ and $2x+1 > 25$

$2x < -24$ $\quad 2x > 26$

$x < -12$ and $x > 13$

No Solution

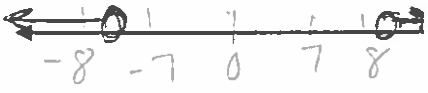
6. $-\left|x-\frac{1}{4}\right|+2 < -6$

$-|x-1/4| < -8$

$|x-1/4| > 8$ "or"

$x-1/4 > 8$ or $x-1/4 < -8$

$x > 8\frac{1}{4}$ or $x < -7\frac{3}{4}$



7. A food manufacturer specifies that every family-size box of cereal should have a net weight of 25 ounces, with a tolerance of 1.2 ounces. Write and solve an absolute value inequality that describes the acceptable net weights for the cereal in a family-sized box, then graph the solution.
(*tolerance* = maximum acceptable deviation of an item from some ideal or mean measurement.)

w = actual weight of cereal box

$|w-25| \le 1.2$ "and"

$w-25 \le 1.2$ and $w-25 \ge -1.2$

$w \le 26.2$ and $w \ge 23.8$

$23.8 \le w \le 26.2$

Quick Check: $2|2x+12| = 8x$ $\quad |2x+12| = 4x$

$2x+12 = 4x$ $\quad 2x+12 = -4x$

$12 = 2x$ $\quad 12 = -6x$

$x = 6$ $\quad x = -2$

Check: $|12+12| = 4 \cdot 6$, $24 = 24$ ✓

Check: $|-4+12| = 4(-2)$, $8 \neq -8$

Do and Discuss:

1. $|2x-5| = 13$

$2x-5 = 13$ $\quad 2x-5 = -13$

$2x = 18$ $\quad 2x = -8$

$x = 9$ $\quad x = -4$

Check: $|18-5| = 13$, $13 = 13$ ✓

Check: $|-8-5| = 13$, $13 = 13$ ✓

2. $3|x+24| = -21x$

$|x+24| = -7x$

$x+24 = -7x$ $\quad x+24 = 7x$

$24 = -8x$ $\quad 24 = 6x$

$x = -3$ $\quad x = 4$

Ch: $3|21| = -21(-3)$, $63 = 63$ ✓

Ch: $3|28| = -21(4)$, $84 \neq -84$

3. $|7x-10|+4 = 8$

$|7x-10| = 4$

$7x-10 = 4$ $\quad 7x-10 = -4$

$7x = 14$ $\quad 7x = 6$

$x = 2$ $\quad x = 6/7$

Ch: $|14-10|+4 = 8$, $8 = 8$ ✓

Ch: $|6-10|+4 = 8$, $8 = 8$ ✓

Solving Absolute Value Inequalities

Inequality	Equivalent Form	Graph
$\|ax+b\| \geq c$	$ax+b \geq c$ or $ax+b \leq -c$	
$\|ax+b\| < c$	$ax+b < c$ and $ax+b > -c$	

$-c < ax+b < c$

*The **inequality** determines what type of compound inequality we write:*

"greater than" → or "less than" → and

Consider $|x| > 5$ and $|x| < 5$

Ex 6: Solve the inequality and graph the solution.

a) $|3x-7| \geq 4$ "or"

$3x-7 \geq 4$ or $3x-7 \leq -4$

$3x \geq 11$ or $3x \leq 3$

$x \geq 11/3$ or $x \leq 1$

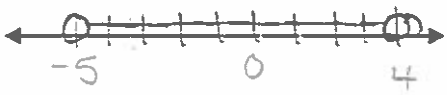
b) $|2x+1|+4 < 13$ "and"

$|2x+1| < 9$

$2x+1 < 9$ and $2x+1 > -9$

$2x < 8$ $\quad 2x > -10$

$x < 4$ and $x > -5$



$-5 < x < 4$